CASTELL

Stadia Slide Rule

No. 1/38 for Surveyors

INSTRUCTIONS

A.W. FABER - CASTELL, STEIN NEAR NUREMBERG

I. Introduction

1. Description of A. W. FABER'S Precision Slide Rule Castell No. 1/38

A.W. FABER'S Precision Slide Rule CASTELL No. 1/38 is a slide rule for general and special computing. Its logarithmic scales permit of all computations which occur in mathematics and their practice, and some special computations in stadia surveying (Tacheometry) can be executed, too.

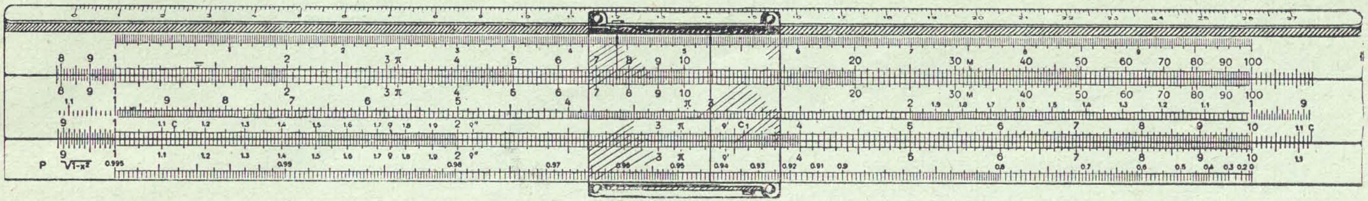


Fig. 1

No. 1/38 Slide Rule consists of the following scales:

1. Main-scales A, B, C, D (= x) and Cr ,
2. Supplementary scales P ($= \sqrt{1-x^2}$), L and the trigonometrical scales,
3. Stadia surveying scales on the reverse of the slide.

2. How to compute with the Slide Rule?

The computation with the slide rule is based on logarithmic laws. It is well known that:

1. The **Multiplication** of two factors is performed by **Adding** their logarithms,
2. The **Division** is performed by **Subtracting** the logarithm of the divisor from the logarithm of the dividend.

Therefore, by using the slide rule every method of computation is replaced by a more simplified operation, and even these simplified operations are spared as they are performed graphically. Owing to this the process of multiplication of two numbers is reduced to adding two lengthes (sections), and the process of division is reduced to subtracting the length (section) of the divisor from the length (section) of the dividend.

The slide rule scale is a graphical representation of **logarithmic** values, as Fig. 2 shows.

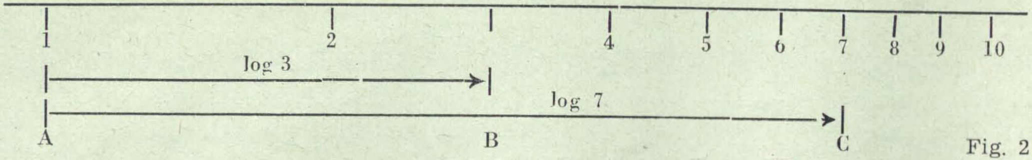


Fig. 2

The number 3 stands at the end of the section of log 3. All logarithmic sections are measured from the left initial mark (point A) of the scale, called the index of the scale. The index is numbered 1, because $\log 1 = 0$.

When the addition and subtraction of the sections have been performed on the logarithmic scales of the slide rule, the result is not the sum and difference of both numbers, but the **product** and **quotient**, as Fig. 3 and 4 show.

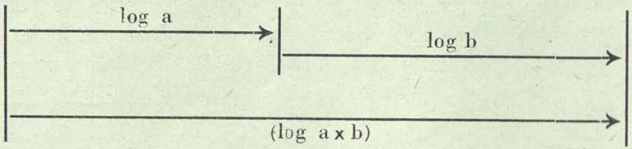


Fig. 3

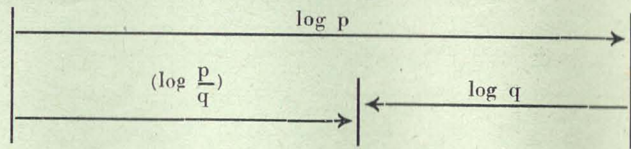


Fig. 4

All other operations performed by means of the logarithmic scales are only variations of these two main problems.

If we wish to compute with the slide rule as shown by Fig. 3 and 4, we have to **"set"** the given numbers on the scales. Therefore we have to understand how to **"read"** the scales. Some practice is required to study the value of the graduations. It is necessary to consider the whole range of the scales and it must be avoided to confuse numbers like 3.04 and 3.4 or 2.14 and 2.18. When the graduations of a section are understood thoroughly, it is suitable to practise the insertion and estimation of the last figures. It is noted that the graduations of the main and CR scales read to three figures. Four-figure readings are obtained by subdividing the spaces between the graduations by eye. Generally this will not cause any difficulty after becoming familiar with the main and CR scales. Judgment and practise will enable an accurate determination of the fourth figure.

Experience has shown that this degree of accuracy is quite sufficient for all simple operations in mathematics.

It is important to note that the decimal point has no bearing upon the slide rule computations. Therefore, numbers like 13.45, 0.1345, 1345, 1.345, are all read as a row of figures 1-3-4-5. Frequently the position of the decimal point may be determined by inspection. In cases of complex calculations then recourse is made to the rule for approximations for positioning the decimal point.

II. Instructions for the Operations

1. The Main Scales

Each slide rule has the upper scales, **A** and **B** and the lower scales, **C** and **D.** Therefore, these are called the **Main Scales** of the rule.

Scales **A and B** are exactly alike and they are divided into 18 parts by primary marks numbered 1, 2, 3, . . . 10, and 20, 30 . . . . 100.

Scales **C and D** are also alike and are divided into 9 parts by primary marks numbered 1, 2, 3, ... 9, 10.

Scales **A and D** are on the **body** of the rule and are, therefore, known as the **Body Scales. B** and **C**, being on the **slide**, are known as the **Slide Scales.**

In addition to these four scales, there is a **reciprocal,** or reversed C scale on the centre of the slide between B and C. This scale, **Cr,** runs from **10 to 1** (Fig. 5) and is marked in RED

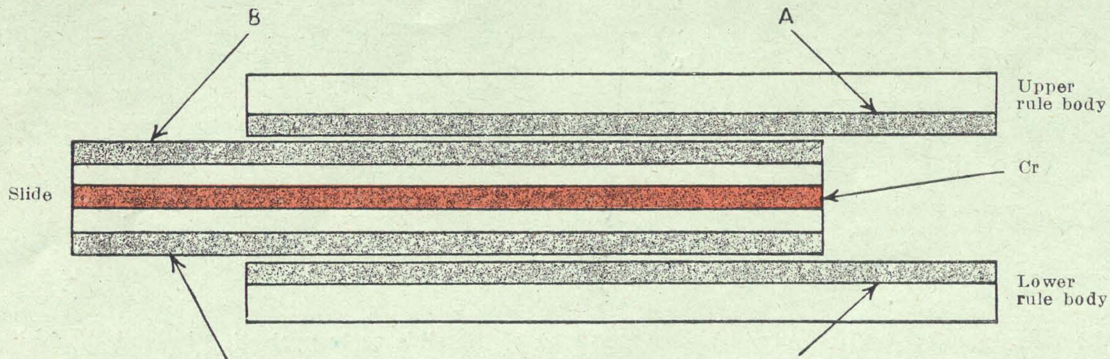


Fig. 5

These five scales are graduated beyond the index at their ends, each being marked in RED.

For all combined operations, consisting of multiplications and divisions only, the three scales **C, D** and **Cr** should be used.

The glass cursor, sometimes called the indicator or rider, has a hairline. With this hair-line of the cursor we can set numbers on the body and the slide scales.

a) Multiplication with the C and D Scales

Either

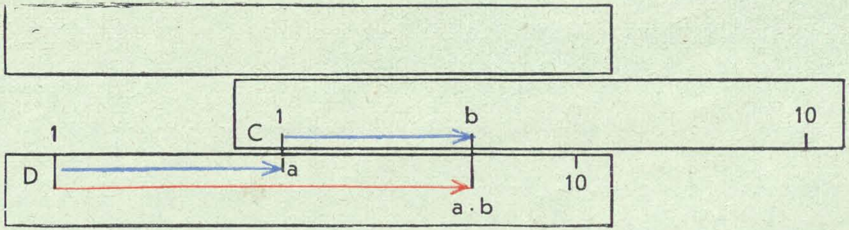
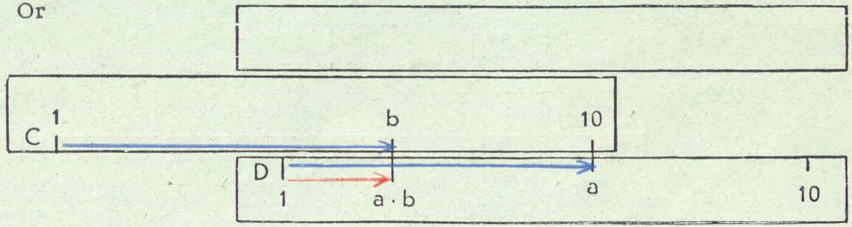


Fig. 6

Or

Fig. 7



a x b

When the second number b of the product is on that part of the slide scale projecting beyond the body, we use the other index of the slide scale, as shown in Fig. 7. Using the right index in place of the left one, and vice versa, is called "Resetting".

Both scales represent a **table**; on scale D we read the numbers which are **a-times** the values of numbers set on scale **C.**

The troublesome resetting of the slide can be avoided by computing with the upper scales A and B, but with less accuracy in the answer for the reason that scales A and B have 18 divisions whereas scales C and D have 9 divisions and the respective length of each scale is the same.

b) **Division with the C and D Scales**

Either

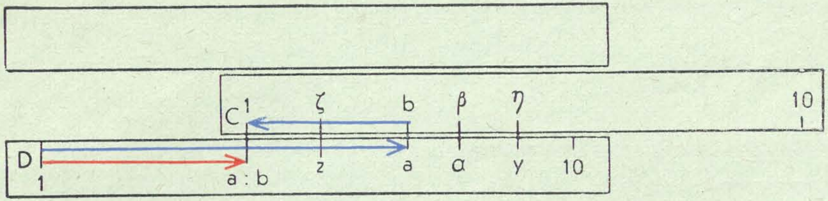
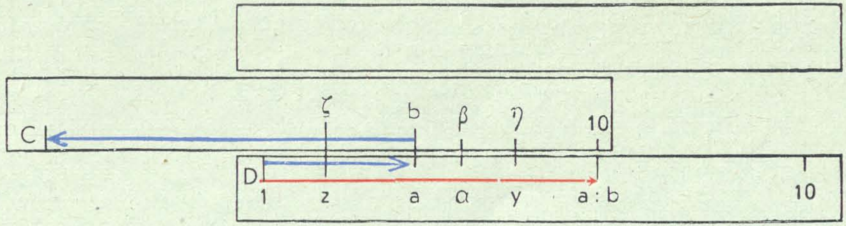


Fig. 8

Or

Fig. 9



The answer can only be read at **that** index of the **C** scale which is inside the rule body.
This setting performs a **table** of all pairs of numbers that have the ratio $\frac{a}{b}$

$$\frac{z}{\zeta} = \frac{a}{b} = \frac{\alpha}{\beta} = \frac{y}{\eta}$$

In this manner all conversions requiring the fourth proportion can be solved, such as:
When metres are set on C, we read yards on D, setting 75 m we read 82 yds.

If $y = \frac{x}{c}$ is to be solved for many values of x, the method shown in Fig. 10 will be the most suitable.

$$y = \frac{x}{c}$$

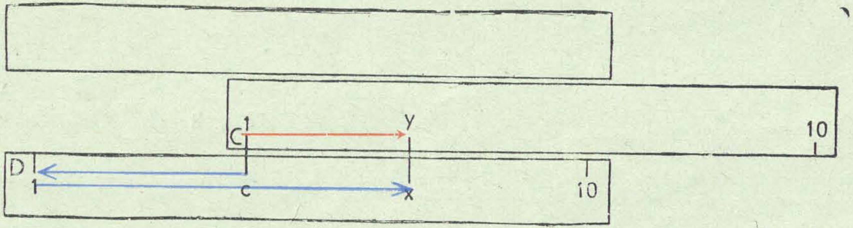


Fig. 10

c) Multiplication and Division by Using Scale Cr

The reversed scale **Cr** (coloured red in order to avoid errors) is a very important scale simplifying many calculations.

Between scales **C and Cr** there is a **reciprocal relationship**, each setting on C being the reciprocal value of that setting exactly above Cr, and vice versa.

Exercises: 30 and 0.0333, 2.5 and 0.4, 125 and 0.008.

If both factors a and b are set in line on D and Cr by means of the hairline, a very convenient method of multiplying is obtained. The answer is always found, either by reading on the left (Fig. 11) or on the right index (Fig. 12).

a x b

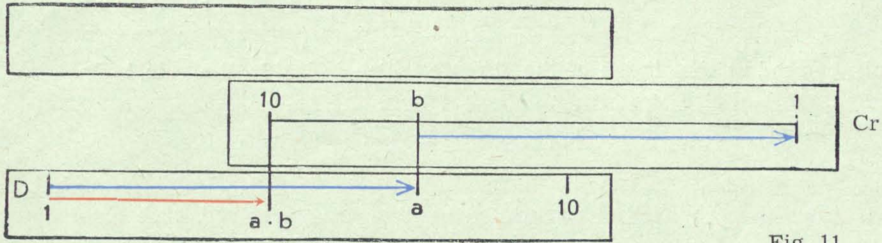


Fig. 11

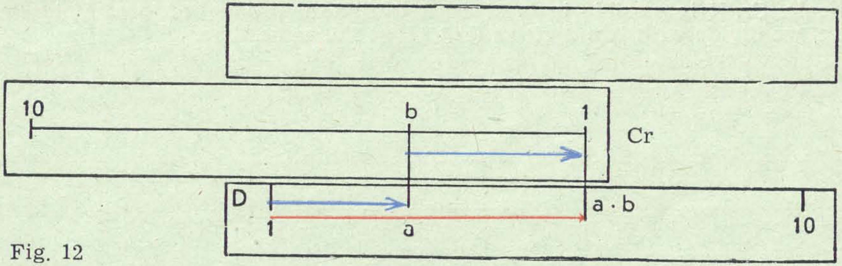
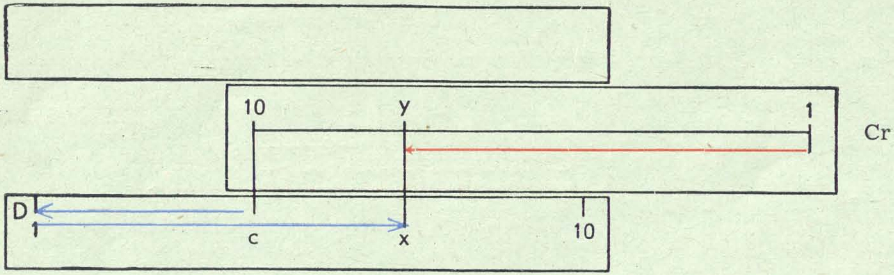


Fig. 12

$$a \times b$$

If $y = \frac{c}{x}$ is to be solved for various values of x, the method shown in Fig. 13 should be employed. With this setting a table is formed at the same time which gives all pairs of numbers having c as their product **(inverse proportion).**



$$y = \frac{c}{x}$$

$$x \times y = c$$

Fig. 13

By this setting we also can find the number c out of all possible factors satisfying the quadratic equation

$$\mathbf{x^2 + s \times x + c = 0}$$

for the sum must be—s.

With the reverse scale, Cr, it is possible in most cases to find the **product of three factors** with o n e setting (Fig. 14). Reversing the procedure, we divide with two divisors (Fig. 15) at the same time.

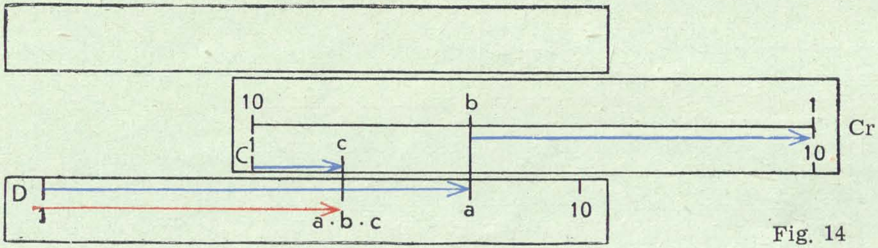
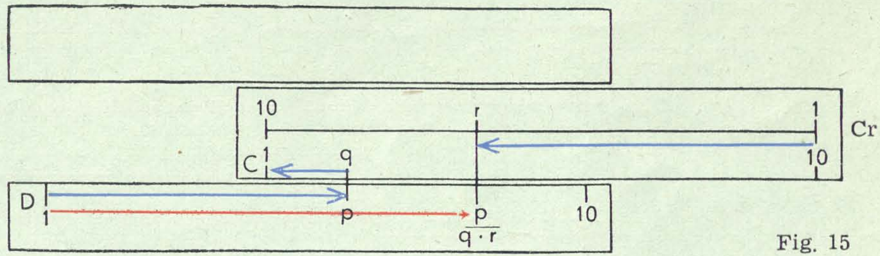


Fig. 14

$$a \times b \times c$$



$$\frac{p}{q \times r}$$

Fig. 15

d) **Squares and Square Roots**

A and B are graduated in a scale of 1 to 2 of the scales C and D. As the scales are logarithmic, the values on the scales C and D are the square roots of the corresponding values exactly lying above on A and B. Conversely the values on the upper scales are the squares of the corresponding values exactly lying below on C and D, because by the logarithmic principle, we have log $x^2 = 2 \times$ log x and log $\sqrt{x} = {}^1/_2$ log x.

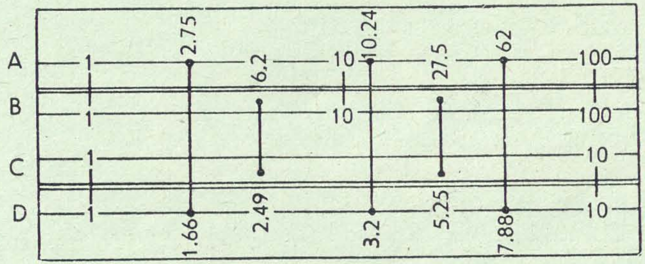


Fig. 16

a^2

$\sqrt{b}$

To find the square root of a given number on scale A, we shall refer to the left-hand part 1—10 as A left and the right-hand part 10—100 as A right. Similar reference is made to the B scale. To find the square root of a number between 1 and 10, set the hairline to the number on A left and read its square root at the hairline on D. To find the square root of a number between 10 and 100, set the hairline to the number on A right and read its square root at the hairline on D. In both cases the decimal point is placed after the first digit. Similar operations are made using B scale and C scale. To find the square root of **any** number, the decimal point is moved an **even** number of places to obtain a number between 1 and 100. Now we can apply the rule mentioned above. The position of the decimal point in the answer will be as half the number of places the decimal point was moved in the original number (to reduce the value of the number to between 1 and 100) but the movement must be in the opposite direction.

Example: $\sqrt{1922} = \sqrt{100 \times 19.22} = 10 \times \sqrt{19.22} = 10 \times 4.38 = 43.8,$

$$\sqrt{0.000\,071} = \sqrt{\frac{71}{1\,000\,000}} = \frac{\sqrt{71}}{1\,000} = \frac{8.43}{1\,000} = 0.008\,43.$$

The upper and lower scales can also be used in combined operations. Many computations are possible, as the following Figures will illustrate.

When the calculation contains a **square,** it is necessary to commence on the **lower** scales to find the answer on an upper scale. There are eight possible.

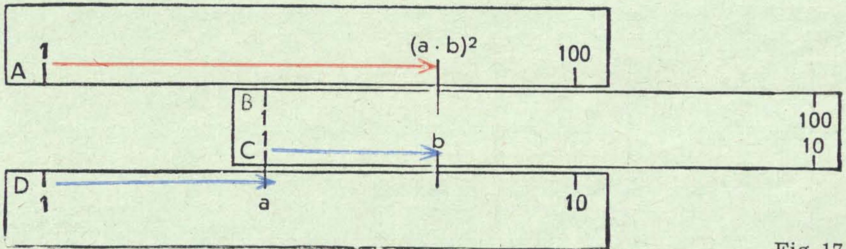
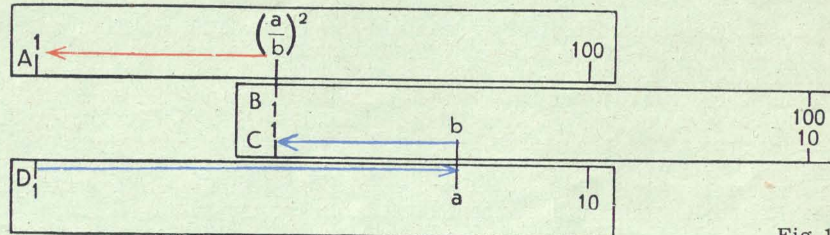


Fig. 17

$$(a \times b)^2$$



$$\left(\frac{a}{b}\right)^2$$

Fig 18

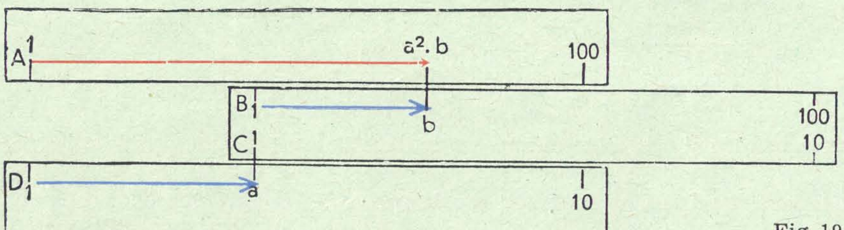


Fig 19

$$a^2 \times b$$

$$\frac{a^2}{b}$$

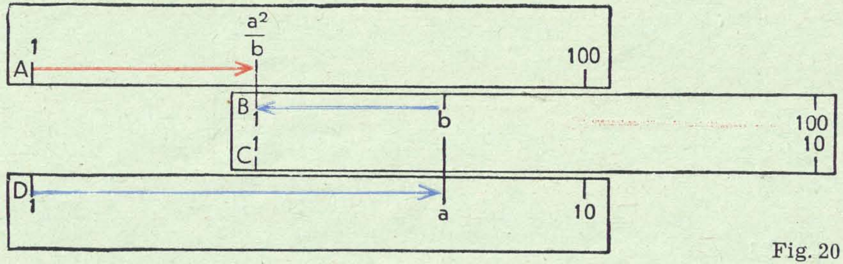


Fig. 20

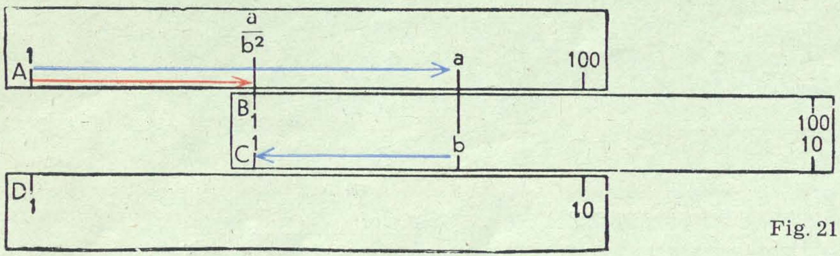


Fig. 21

$$\frac{a}{b^2}$$

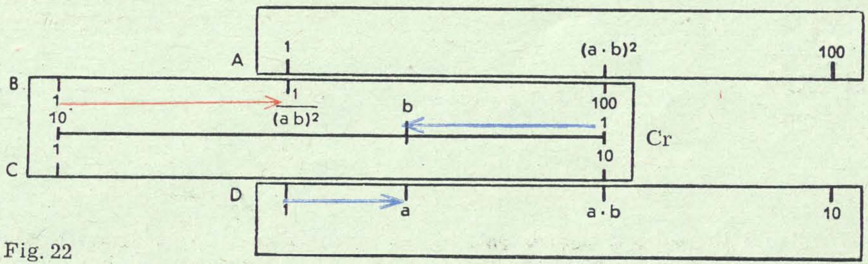


Fig. 22

$$\frac{1}{(a \times b)^2}$$

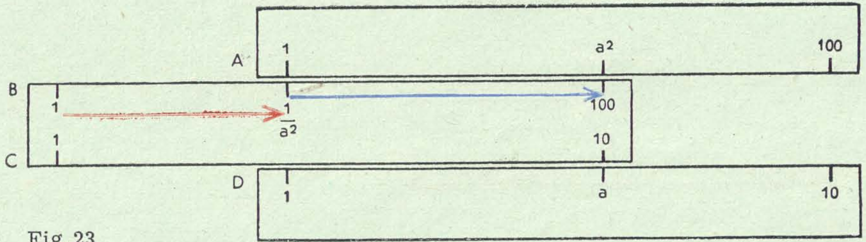


Fig. 23

$$\frac{1}{a^2}$$

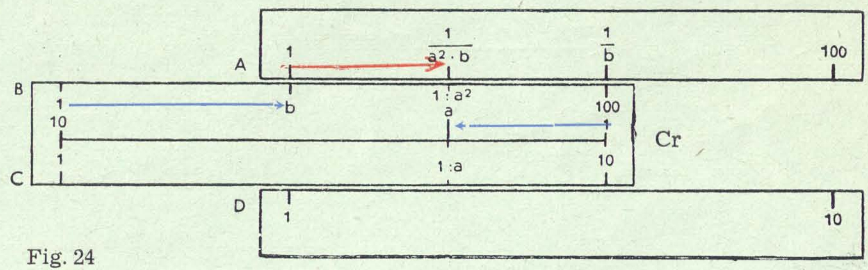
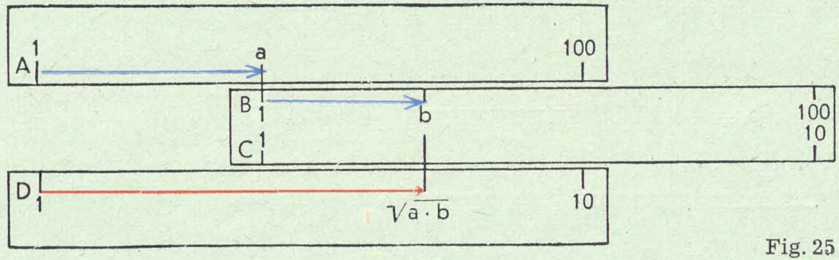


Fig. 24

$$\frac{1}{a^2 \times b}$$

Should there be a **square root** in the calculation, it is necessary to **start** on the **upper** scales, to find the root on a lower one. Care must be taken to set the number on the correct half of scale **A** or **B.**



$\sqrt{a \times b}$

Fig. 25

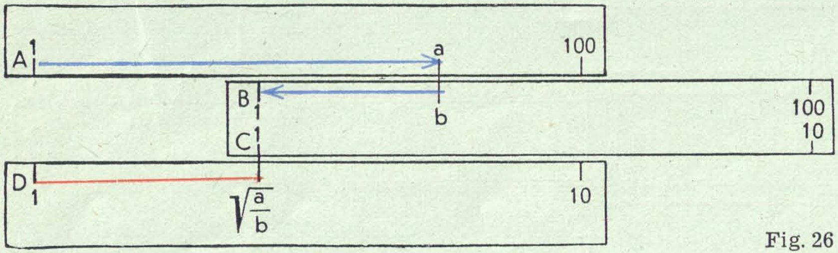
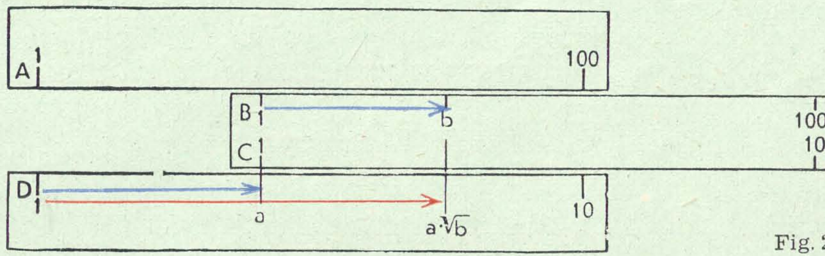


Fig. 26

$\sqrt{\dfrac{a}{b}}$



$a \times \sqrt{b}$

Fig. 27

$$\frac{a}{\sqrt{b}}$$

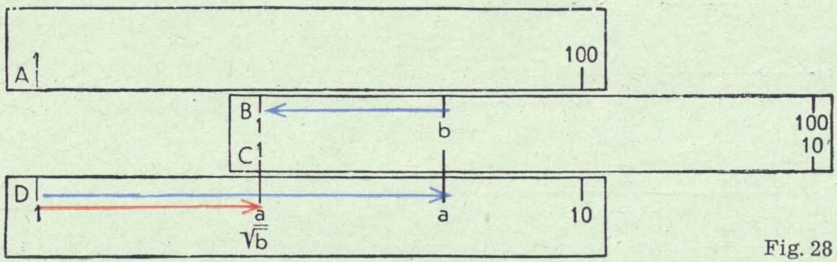


Fig. 28

$$\frac{\sqrt{a}}{b}$$

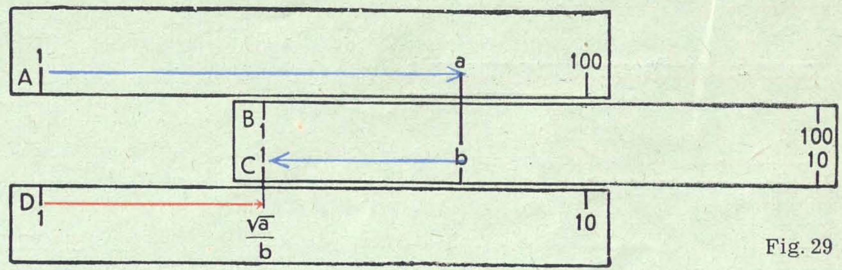


Fig. 29

$$\frac{1}{\sqrt{a \times b}}$$

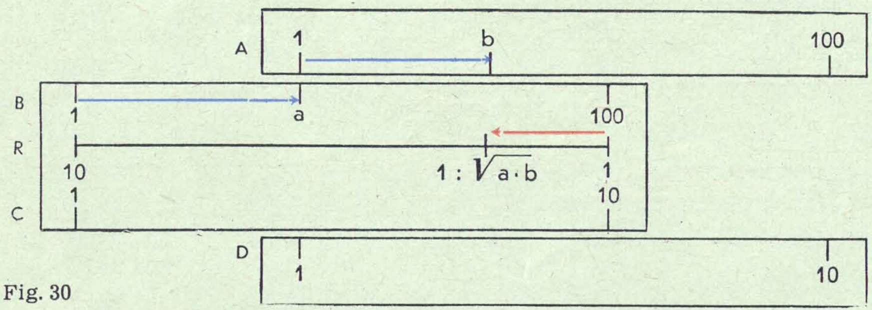


Fig. 30

$$\frac{1}{a \times \sqrt{b}}$$

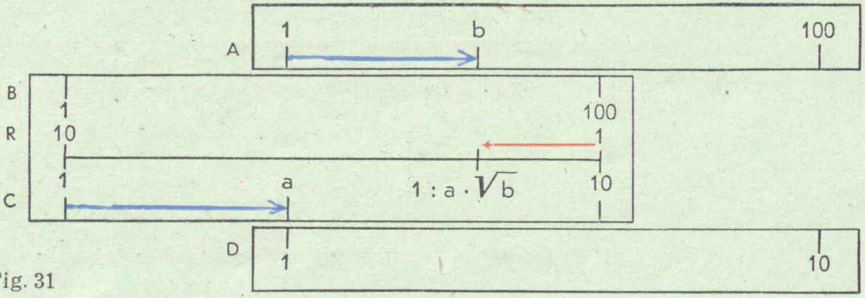


Fig. 31

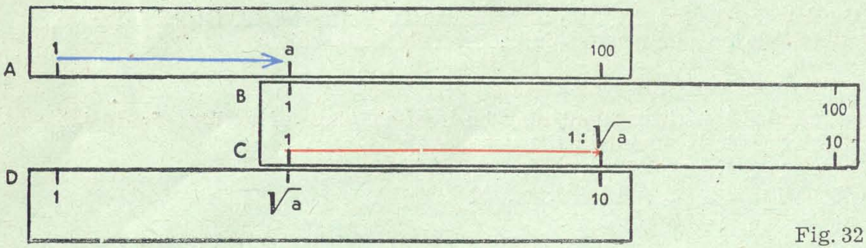


Fig. 32

$$\frac{1}{\sqrt{a}}$$

2. *The Supplementary Scales*

Additional scales are provided to facilitate computations other than multiplications, divisions, squares, and square roots. The following scales are used in connection with the **D** scale:

1. **The uniformly graduated Scale L** on the upper edge of the rule body, above the **A** scale, for reading the common logarithms (to base 10) (Fig. 33)

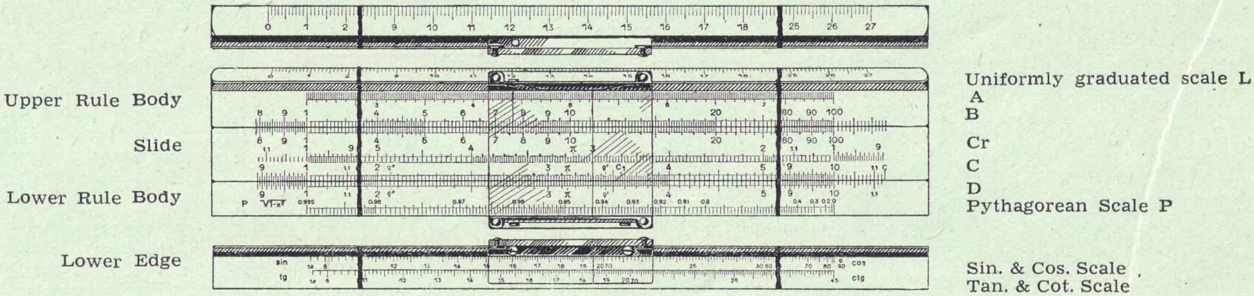


Fig. 33

2. The **pythagorean scale P** ($\sqrt{1-x^2}$) on the lower edge of the rule body below **D.** Its use is explained later on.
3. The scale for the four **trigonometrical functions** on the small side of the rule body below.

(a) The uniformly graduated scale

This scale works with scale **D** for reading common logarithms and may be used in place of a **three-figure table of logarithms.** We only read the mantissa; the characteristic is found in the usual way.

Example: log 52 = 1.716
(Fig. 34) log x = 3.574, x = 3750

log x

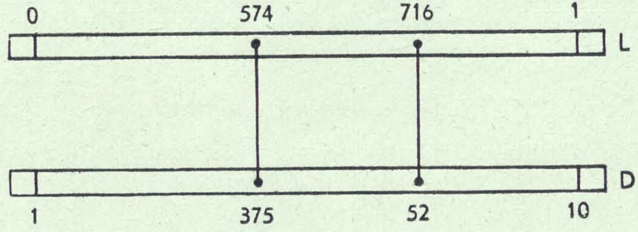


Fig. 34

(b) The Pythagorean Scale P

This scale represents the function $y = \sqrt{1-x^2}$ and is employed in connection with scale **D** (= x). On **D** the numbers are to be read as 0.1 to 1. The Pythagorean Scale is running in the opposite direction and is marked in RED. See also pages 20 and 21.

Example: $x = 0.8$ $y = 0.6$ $\sin \alpha = 0.134$ $\cos \alpha = 0.991$
(Fig. 35)

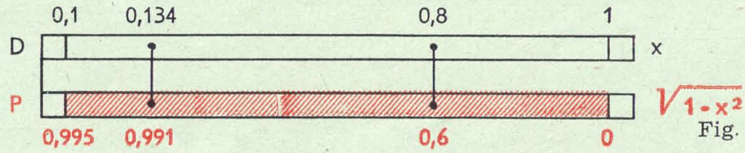


Fig. 35

$\sqrt{1-x^2}$

(c) The Trigonometrical Scales with secondary, decimally graduated marks

Use of the scales as tables.

Reading the sin-cos scale from left to right, with the **Black Numbers,** we obtain a **Sine Table** on Scale **D (black).** Reading the **Red Numbers** from right to left, we obtain another **Sine Table** on **P (red).**

For smaller angles the first procedure is preferable; for larger angles the second one. In Fig. 36, $\sin 76^0$ may be read as 0.97 on scale D, but on P more exactly as 0.9703.

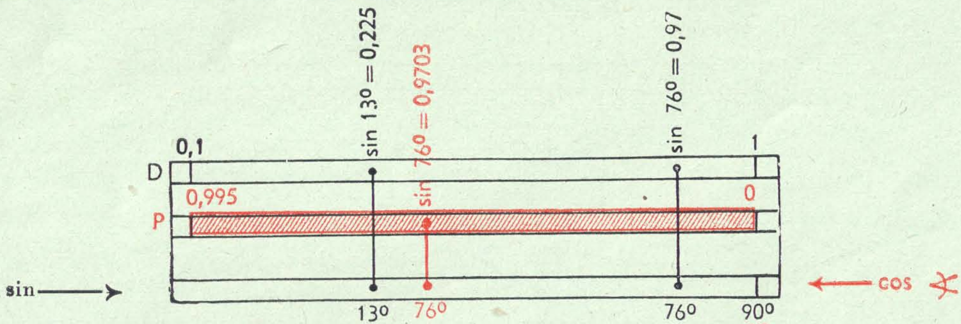
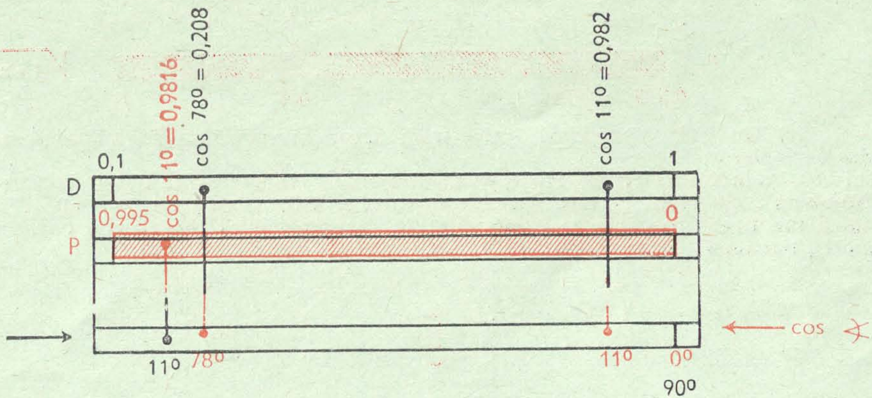


Fig. 36

sin α

The numbers of the sine scales represent angles. In order to set the hairline on the given value of an angle on the sine scales, it is necessary to have in mind the number associated with the smallest space of the graduation under consideration.

Reading the sin-cos scale from right to left, with the **Red Numbers,** we obtain a **Cosine Table** on **D (black).** Reading the **Black Numbers** from left to right, we obtain another **Cosine Table** on **P (red).** For larger angles the first procedure is preferable, for smaller angles the second one. In Fig. 37, cos 11^0 may be read on D as 0.982, but on P more accurately as 0.9816.



cos α

Fig. 37

These different ways of reading can easily be remembered by the following rule: Same colour for sin, different colour for cos.

Reading the tan-cot scale from left to right, with the **Black Numbers,** we obtain a **Tangent Table** on **D (black).** Reading the **Red Numbers** from right to left, we obtain a **Cotangent Table** on **D (black).** It would appear as if only tangents of angles smaller than 45^0 and cotangents of angles larger than 45^0 can be read. But as tan α and cot α are reciprocal values, the use of scale **Cr** permits all values to be read, shown in the examples of Fig. 38. The procedure is summarised in the following:

Tangents	smaller than 45° larger than 45°	black numbers and D or C red numbers and Cr
Cotangents	smaller than 45° larger than 45°	black numbers and Cr red numbers and D or C

As a rule, reading **Tangents** we have **same colours,** reading **Cotangents** we have **different colours.**

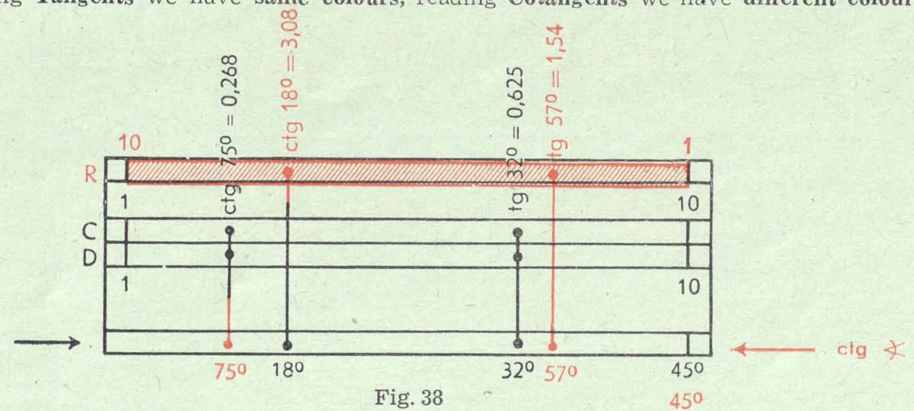


Fig. 38

tan α

cot α

Functions of small angles

The trigonometrical functions can be read in this way down to 5.7°, for sin 5.7° $\approx$ tan 5.7° $\approx$ 0.1. (the symbol $\approx$ means "approximately equals")

The following relationship can be used for angles smaller than 5.7°:

$$\sin \alpha \approx \tan \alpha \approx \operatorname{arc} \alpha \approx \frac{\alpha^0}{\varrho^0} \approx \frac{\alpha^0}{57{,}3}$$

The mark ϱ^0 has been engraved at 5 7 3 on the C and D scales; it is set as illustrated in Fig. 39.

$$\sin 3^0 \approx \tan 3^0 \approx \text{arc } 3^0 \approx 0.0524.$$

The error is less than 0.25 %.

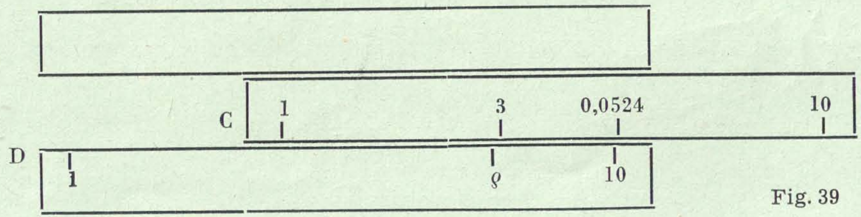


Fig. 39

The Cosine of a small angle is found in the following manner:

$$\cos \alpha = 1-2 \sin^2 \frac{\alpha}{2} \approx 1 - \frac{(\text{arc } \alpha)^2}{2} \approx 1-1.52 \times 10^{-4} \alpha^2$$

The Cotangent of a small angle:

$$\cot \alpha = \frac{1}{\tan \alpha} \approx \frac{1}{\text{arc } \alpha} = \frac{\varrho^0}{\alpha} = \frac{57.3}{\alpha}$$

The scale of angles in "grad"-division (400 ^{g})

Rule 1/38 is available with trigonometrical scales in centesimal graduation (100 ^{g} to the quadrant). The foregoing examples can be applied to this scale without difficulty.

Examples: sin 73 g = 0.911, more exactly 0.9114
sin 14 ^{g} = 0.218
cos 14 g = 0.976, more exactly 0.9759
cos 81 ^{g} = 0.294

tan 32 ^{g} = 0.549
tan 57 g = 1.248
cot 18 ^{g} = 3.44
cot 75 ^{g} = 0.414

Mark

ϱ^g. $\varrho^g = 0.01\ \varrho^c = 0.0001\ \varrho^{cc} = 63.66$

The trigonometrical scales extend only as far as 6.38 g, for sin $6.38^g \approx$ tan $6.38^g \approx 0.1$.
The mark ϱ is provided for reading the functions of smaller angles. It is engraved on C and D, between 63 and 64. For such small angles, the trigonometrical functions, sine and tangent, are almost identical with the arc.

Example: sin $0.17^g \approx$ tan $0.17^g \approx$ arc $0.17^g = 0.002\ 67$.

Place hair line to mark ϱ on D, draw 17 of C under the hair line, opposite the right index of D. read the value of the function. Sometimes the reading is to be made opposite the left index

Example: sin $0.0085^g \approx$ tan $0.0085^g \approx$ arc $0.0085^g = 0.000\,1336$.

Place hair line to mark ϱ on D, draw 85 of C under the hair line and opposite the left index of D, read the value of the function on C.

Computing with Trigonometrical Scales

To change from the sine of an angle to the cosine (or vice versa), the value of the angle need not be read. On scales **D** and **P** these pairs of values are in line. Also by changing from tangents to cotangents the reading of the angles is not necessary as the corresponding values are in line on **C** and **Cr.** Only when changing from sines or cosines to tangents or cotangents, the value of the angle is to be read.

As the functions can be read on **D** or **Cr,** combined operations involving multiplication or division can be performed in continuation. Readings on **P,** however, must be transferred to the main scales.

3. *Special Scales for Stadia Surveying (Tacheometry)*

For reducing inclined stadia readings the terms for

Horizontal Distance E $\cos^2$ (Hor. Dist.) HD
Horizontal Correction E $(1 - \cos^2)$ (Hor. Corr.) HC
Difference in Elevation E sin cos (Diff. Elev.) DE

are to be computed. For this purpose three special scales on the reverse of the slide are used. The slide is removed from the body of the rule and then replaced with the special scales showing between the main scales "A and D".

The following special scales are on the reverse of the slide:

1. A scale giving the values of sin α and cos α. The sin scale is positioned at the edge of the slide adjacent to scale D and ranges from 0.5^0 to 6^0 (0.6^g to 7^g). This scale is continued above the scale (i. e. from 0.5 to 6^0) but only along the left half of the slide and ranges from 5^0 to 20^0 (respect. 6^g to 22^g).
2. A short scale of values of cos $^2\alpha$, on the right side in the middle of the slide, ranging from 20^0 to 5^0 (resp. 25^g to 5^g).
3. A scale of values of 1 — cos $^2\alpha$ at the upper edge of the slide. This scale consists also of two parts; the upper part lies exactly opposite the scale A, ranging from 5.7^0 to 20^0 (resp. 6^g to 22^g). The second part lies below the other one, ranging from 1.85^0 to 5.7^0 (resp. 2^g to 7^g).
   The term E to be set as one of the factors, must be computed to E = c + kl.

Here is:

1. c the stadia constant ($0 \leq c < 60$ cm), see graph at reverse of rule body,
2. k the multiplying factor, in most theodolites or levels $99.50 < k < 100.50$
   rarely $199.00 < k < 201.00$

   For k a round value of 100 (or 200) is desirable. In the other cases E is suitably computed with the formula

$$E = 100\,l + (k - 100)\,l + c$$
$$\text{resp. } E = 200\,l + (k - 200)\,l + c$$

   To the value 100 l (or 200 l) computed mentally, the small expression (k — 100) l + c or (k — 200) l + c is added. The reverse of the rule body bears a sufficiently accurate table with two entries for k and for 100 l and 200 l. The desired value of (k — 100) l or (k — 200) l is to be found in the column headed by the corresponding value of 100 l or 200 l, and in the row having as its first or last entry the multiplying factor k of the theodolite or level. Every theodolite or level corresponds with a certain row of that table.
3. l is the rod interval between the stadia hairs. The rod-readings can be taken when the rod is vertical or horizontal.

(a) Computing with vertical rod

The difference in elevation between two points is computed with the term E sinα cosα and their horizontal distance with the term E cos $^2\alpha$. The instructions and examples given in the subsequent pages explain the uses of the stadia scales:

1. The process of multiplication is performed by the same manner as previously explained, adding the sections corresponding to the logarithmic values of E and sin α cos α.
2. The stadia scales are to be applied in connection with the lower scale D.
3. The space between any two primary marks as α^0 (α^g) of the stadia scales is graduated into decimal parts of 0.05^0 (0.05^g). For more exact computation the estimation of fifth parts of the smallest space between the secondary marks may be set ($= 0.01^0$ or 1^c) or the estimate of third parts ($= 1'$).
4. The decimal point must be positioned by the computer by approximating the answer. Generally the position of the decimal point may be determined by inspection — also see examples below.

Computation of the Difference in Elevation E sin α cos α

For Estimating the position of the decimal point:

$$\sin 6^0 \cos 6^0 \approx 1/10$$
$$\sin 0.6^0 \cos 0.6^0 \approx 1/100$$

Performing the multiplication E sin α cos α the slide can be set in two ways, namely

a) Slide to left see multiplication 3×4 on C and D
b) Slide to right see multiplication 2×4 on C and D

Slide to left, setting the right index

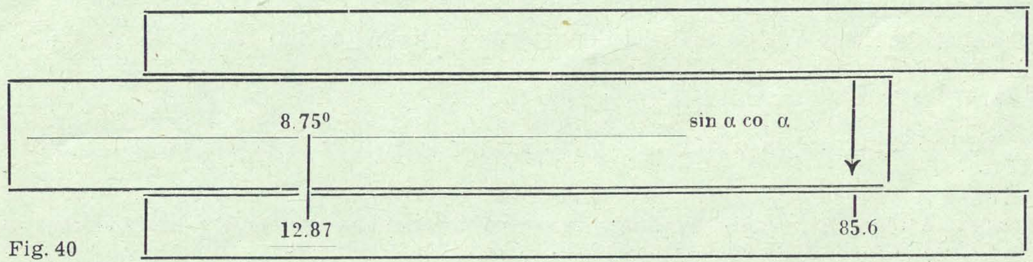


Fig. 40

Examples for graduation 360°

Given: E = 85.6 ft and α = 8.75°

Set right index of the stadia slide to 85.6 on D (the vertical mark running over the whole slide on its right end), place hairline of the cursor to 8.75° on the sincos-scale, at the hairline read 1287 on D. The difference in elevation must result more than 10 % of E, therefore 12.87 ft.

Other exercises:	E = 246.5 ft, α = 3.1°,	answer 13.31 ft.	E = 337.5 ft, α = 4,4°	answer 25.8 ft.
	E = 192.1 ft, α = 5.16°,	„ 17.21 ft.	E = 137.0 ft, α = 5°	„ 11.89 ft.

Slide to right, setting the left index

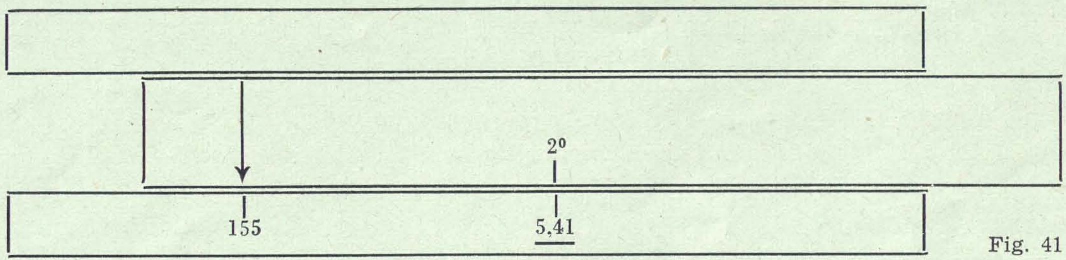


Fig. 41

It may happen that the value α is on the part of the slide projecting to left beyond the body; the slide is to be reset to right, using now the left index.

Given: E = 155 ft, and α = 2°

Set the left index of the slide to 155 on D, place hairline to 2° off the sincos-scale (opposite scale D), at the hairline read 541 on D. The difference in elevation is much smaller than 10 % of E, but more than 1 % of E, therefore the result is 5.41 ft.

Given: E = 182.4 ft and α = 6.07°

Set the left index to 182.4 on D, place hairline to 6.07° of the sincos-scale, at the hairline read 1918 on D. The result must be nearly 10 % of E, therefore the difference in elevation is 19.18 ft.

Further exercises:	E = 275.3 ft	$\alpha = 1.72^0$	answer 8.26 ft	E = 192.1 ft	$\alpha = 5.16^0$	answer 17.21 ft
	E = 61.5 ft	$\alpha = 7.15^0$	„ 7.6 ft	E = 137.0 ft	$\alpha = 5^0$	„ 11.89 ft
	E = 208 ft	$\alpha = 2.05^0$	„ 7.44 ft			

Examples for "grad"-graduation 400 ^{g}:

E = 137 ft	$\alpha = 5^g$	difference in elevation 10.72 ft			
E = 85.6 ft	$\alpha = 8.75^g$	„ „ „ 11.62 ft	E = 275.3 ft	$\alpha = 1.72^g$	difference in elevation 7.43 ft.
E = 246.5 ft	$\alpha = 3.1^g$	„ „ „ 11.98 ft	E = 61.5 ft	$\alpha = 7.15^g$	„ „ „ 6.85 ft
E = 192.1 ft	$\alpha = 5.16^g$	„ „ „ 15.50 ft	E = 208 ft	$\alpha = 2.05$ g	„ „ „ 6.69 ft
E = 337.5 ft	$\alpha = 4.4^g$	„ „ „ 23.25 ft	E = 85.6 ft	$\alpha = 12.2$ g	„ „ „ 16 ft

Computation of the horizontal distance E cos$^2\alpha$

For the process of this multiplication, two methods are possible:

a) Computing with the small auxiliary scale at the right side of the slide. By using this scale, only the **right** index of the slide must be set.

Given: E = 81.5 ft and $\alpha = 16.5^0$

Set the **right** index to 81.5 on D (pushing the slide somewhat to left),
place hairline to 16.5^0 on the cos^2-scale, at the hairline read 74.9 ft on D.
The position of the decimal point is clear, because the horizontal distance is always the shorter.

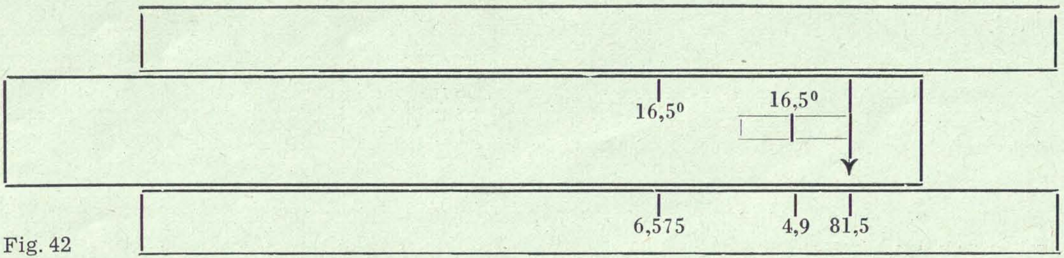


Fig. 42

Other exercises:	E = 66.5 ft	$\alpha = 12^0$	horizontal distance: 63.6 ft
	E = 354 ft	$\alpha = 9^0$	„ „ 345.5 ft
	E = 137 ft	$\alpha = 5^0$	„ „ 136 ft

This first method is not too satisfactory because α is always a small angle, $\cos\alpha$ lies close below 1, and the value of $\cos^2$, which indicates the horizontal distance, is also very close to the end mark 1 of the slide.

b) Therefore it is easier to read the **correction** of the inclined distance to the horizontal (the value of the shortening of E) than the horizontal distance. For this purpose the two scales $1\text{-}\cos^2\alpha$, engraved on the upper edge of the slide, are employed. To obtain the Horizontal Correction HC for the example in Fig. 42,
place hairline (at the same setting of the right index) to 16.5° on the top scale of $1\text{-}\cos^2$,
at the hairline, read the value of shortening 6.58 ft on D,
thus obtaining a more exact result of the horizontal distance which works out to 74.92 ft.

The placing of the decimal point can be found at once by reading the rough value of the horizontal distance by means of $\cos^2$-scale or by the following estimation:

for α = 10° the value of shortening amounts to nearly 3%
= 6° „ „ „ „ „ „ „ 1%
= 2° „ „ „ „ „ „ „ 1‰

For the second example mentioned above the shortening results to 2.87 ft, and therefore the more exact horizontal distance is 63.62 ft. The next following exercise can only be solved by setting the **left** index to 354 on D, the value of shortening is read 8.66 ft, and the horizontal distance is therefore 345.34 ft. For reading the rough and the exact answer, two settings are to be made. The angle α = 5° in the last example has, by this method, to be set on the lower part of the two scales $1\text{-}\cos^2$. The shortening is to be read 1.04 ft and therefore the more exact horizontal distance is 135.96 ft.

Other exercises:

a) By setting the right index, the rough horizontal distance and the exact value of shortening can be read

E = 85.6 ft	α = 8.75°	1.98 ft	83.62 ft
E = 192.1 ft	α = 5.16°	1.55 ft	190.55 ft
E = 61.5 ft	α = 7.15°	0.95 ft	60.55 ft

b) Two settings (left and right index) are required.

E = 182.4 ft	α = 6.07°	2.04 ft	180.36 ft.

If the vertical angles are smaller than 5°, only the more exact value of shortening by means of the scale $1\text{-}\cos^2$ can be read, as shown in the following examples:

E = 155 ft	α = 2°	0.19 ft	154.81 ft
E = 246.5 ft	α = 3.1°	0.72 ft	245.78 ft

E = 208 ft	$\alpha = 2.05^0$	0.27 ft	207.73 ft
E = 275.3 ft	$\alpha = 1.72^0$	0.28 ft	275.02 ft
E = 337.5 ft	$\alpha = 4.4^0$	1.985 ft	335.52 ft

For given E and α computing both the difference in elevation and the horizontal distance, almost the two answers with one setting can be read. Often the rough value of the horizontal distance for the positioning of the decimal point can be found by this one setting, too.

Exercises for grade-graduation 400^g

E = 182.4 ft	$\alpha = 6.07^g$	shortening	1.65 ft	distance	180.75 ft
E = 155 ft	$\alpha = 2^g$	„	0.15 ft	„	154.85 ft
E = 246.5 ft	$\alpha = 3.1^g$	„	0.58 ft	„	245.92 ft
E = 208 ft	$\alpha = 2.05^g$	„	0.22 ft	„	207.78 ft
E = 137 ft	$\alpha = 5^g$	„	0.84 ft	„	136.16 ft
E = 192.1 ft	$\alpha = 5.16^g$	„	1.26 ft	„	190.84 ft
E = 61.5 ft	$\alpha = 7.15^g$	„	0.77 ft	„	60.73 ft
E = 337.5 ft	$\alpha = 16.2^g$	„	21.4 ft	„	316.1 ft
E = 85.6 ft	$\alpha = 8.75^g$	„	1.61 ft	„	83.99 ft

Method for very small angles

The two stadia scales range from 0.5^0 (0.6^g) resp. from 1.85^0 (2^g) to 20^0 (22^g resp. 25^g). Larger angles are seldom taken, but angles smaller than 1.85^0 (2^g) are commonly encountered in tacheometry. In these cases the scale 1 — $\cos^2$ cannot be used. For angles smaller than 0.5^0 (0.6^g) the sincos-scale also is not suitable. For these small angles the method mentioned above is applied, and it is necessary to convert the degrees into minutes. The accuracy of the slide rule allows the following approximation for small angles:

a) 360^0 graduation. $\cos\alpha \approx 1$ and $\sin\alpha \approx \text{arc}\,\alpha ; = \text{arc}\,\alpha' ; \dfrac{\alpha' \times \pi}{180 \times 60} = \dfrac{\alpha'}{3438} = \dfrac{\alpha'}{\varrho'}$

The value 3438 is called ϱ' and the mark ϱ' is engraved on the lower scales C and D.
The stadia reduction formulas are simplified as follows: Difference in elevation E $\sin\alpha\cos\alpha \approx \dfrac{E \times \alpha'}{\varrho'}$

Horizontal correction E $(1 - \cos^2\alpha) \approx 0$

For this computation the stadia scales are not necessary.

Example: E = 97.4 ft, $\alpha = 1.8^0 = 108'$

Place hairline to 974 on scale D, and next position the mark ϱ' on scale C under the hairline. Then opposite 108 on C read 306 on D.

For positioning the decimal point, an estimation will be made: DE at $0.6^0 \approx 1\,\%$ E, in e. g. $3\,\%$ E, therefore 3.06 ft. The shortening is rough $1\,‰$ E $\approx$ 0,1 ft, then the horinzontal distance is 97.3 ft. Computed with logarithms, we obtain 3.0579 ft and 97.304 ft.

Example: E = 173.5 ft α = 23'

Place hairline to 173.5 on D, and next position mark ϱ' of C under the hairline. Then opposite 23 on C read 1161. By an estimation ($2^0 \equiv 1$ ‰ E), the decimal point is placed 1.161. The shortening of the horizontal distance can be spared

Computed with logarethms we obtain 1.1608 ft and 173 · 492 ft.

b) 400 g graduation

$$\text{arc } \alpha^g = \frac{\alpha^g \times \pi}{200^g} = \frac{\alpha^g}{63.662} = \frac{\alpha^g}{\varrho^g}$$

The value 63.66 is called ϱ^g and the mark ϱ^g is engraved on the lower scales C and D.

The operation and the formulae are the same as for 360^0.

Examples:
E = 97.4 ft $\alpha = 1.8^g$ Diff. in elevation 2.75 ft
E = 173.5 ft $\alpha = 23^c$ Diff. in elevation 0.675 ft.

Computed with logarithms: 2.75245 and 0.6268 ft.

b) Computing with horizontal rod

In Tacheometry of a higher degree of precision, a horizontal rod is used and therefore the following formulae are applied:

Horizontal distance E cos α

Horizontal correction = $2 \text{ E} \sin^2 \frac{\alpha}{2}$

Difference in elevation = E sin α

The problems are solved by means of the mark ϱ' (ϱ^g). The special scales for Tacheometry are not necessary.

Exercise: E = 175.7 ft. $\alpha = 2^0\ 14'$

$$\frac{\alpha}{2} = 1^0\ 07' = 67'$$

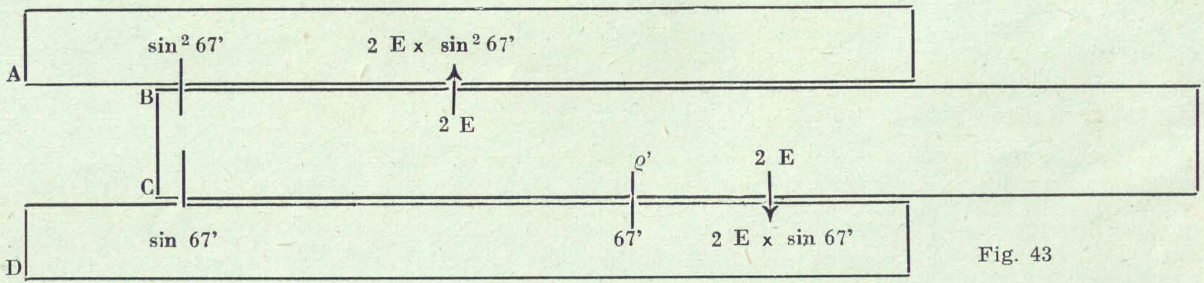


Fig. 43

Place hairline to 67 on scale D, and next position mark ϱ' on scale C under the hair line. Opposite 1 (left index) on scale C is the value of sin 67' on scale D and therefore opposite 1 (left index) on scale B the value of $\sin^2 67'$ on scale A. But these two values will not be read.

Now place hairline to 2 E = 351.4 on scale B, at the hairline read the horizontal correction 133 on scale A. The estimation of the shortening gives 1 ‰ for $\alpha = 2^0$, therefore the result is 0.133 ft.

With the same setting the difference in elevation is found. Since α usually is a small angle, $\sin \alpha$ is nearly the same as $2 \sin \frac{\alpha}{2}$, therefore difference in elevation $= 2\,E \sin \frac{\alpha}{2}$. Place hairline to 351.4 on scale C, and at the hairline read 685 on scale D.

An estimation of the approximate answer gives 3 % E for difference in elevation, therefore the result is 6.85 ft.

c) Computation of the correction for curvature of the earth and refraction

Should differences in elevation over very long sights be determined trigonometrically, the combined correction for curvature of the earth and for refraction has to be applied. This correction of the level surface is computed by formulae

$$V_H \frac{1-k}{2\,r} \cdot S^2$$

of which k is the coefficient of refraction
r is the radius of the earth
s is the distance between the two points

With $k = 0.15$ and $r \approx 3960$ miles $\approx 20\,909\,000$ ft $\approx 20\,239\,000$ Cape ft the value of the term $\frac{1-k}{2\,r}$ is

for V_H in ft and s in miles: 0,567
s in 1000 ft: 0,0203
for V_H in Cape ft and s in miles: 0,548
s in 1000 Cape ft: 0,0210

Examples:

1) The given distance is 16.85 miles. Find the value of V_H. Set index of C to 16.85 on D, push hairline to 567 (to 548) on B, at the hairline read 161 ft (156 Cape ft) on A.

2) The heights of two signals are 46.6 Cape ft and 67.2 Cape ft. How large is the horizontal distance for simultancous observing the tops of the two signals?

a) Push hairline to 46.6 (not 4.66) on A, draw 210 of B under the hairline, at the index of C read 47.1 on D.

b) Push hairline to 67.2 (not 6·72) on A, draw 210 of B under the hairline, at the index of C read 56.6 on D. The whole distance is now: 47 100 + 56 600 = 103 700 Cape ft.

Compudet with the value 0,548 the result of the distance in miles is: 9.22 + 11.07 = 20.29 miles.
For estimating the approximate answer: For horizontal distance: 10 miles is V_H: 56.7 ft (54.8 Cape ft)

	10 000 ft	2.03 ft
	10 000 Cape ft	2.10 Cape ft
70 200 ft	≈ 13,3 miles	100 ft
69 000 Cape ft	≈ 13,5 miles	100 Cape ft

Which Parts of the Slide rule can be repaired?

The cursor can be replaced. Dealers generally have cursors in stock. When buying, the number shown in the centre of the slide rule, below the slide, should be given. The table of constants on the underside of the slide rule can also be replaced. It is, however, not possible to replace slides or celluloid strips showing the logarithmic graduations, as the graduations on the rule body and slide must be engraved in one turn of work in order to coincide.

Treatment of Slide Rules

The Slide Rule should not be exposed to the sun's rays or to strong variations of temperature or moisture; it should be kept in a dry and cool place. It is advisable to clean the celluloid parts from time to time with a soft rag, slightly moistened with petrol or gasoline, or with a soft rubber free from glass. Never use alcohol; same would dissolve the celluloid and the colour of the graduation. In order to ensure good working of the slide, moisten its edges from time to time with a THIN coating of pure vaseline. Never put a cigarette on a slide rule. Burnt parts cannot be replaced.

1/738e