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how to use... trig SLIDE RULES

PICKETT

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Price 50 Cents

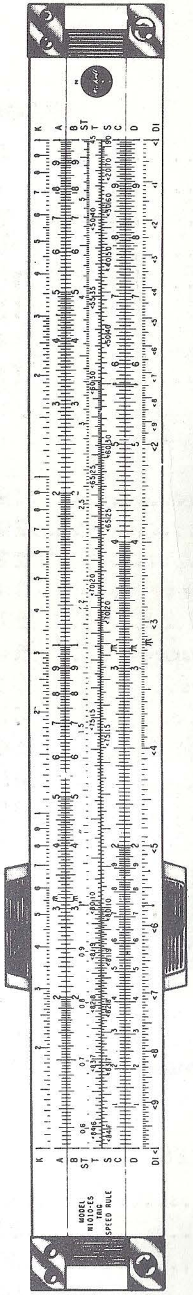
PREFACE

A computer who must make many difficult calculations usually has a slide rule close at hand. In many cases the slide rule is a very convenient tool because by means of a few simple adjustments the calculations can be carried through and the result obtained. One has only to learn to read the scales, how to move the slide and indicator, and how to set them accurately, in order to be able to perform long and otherwise difficult calculations.

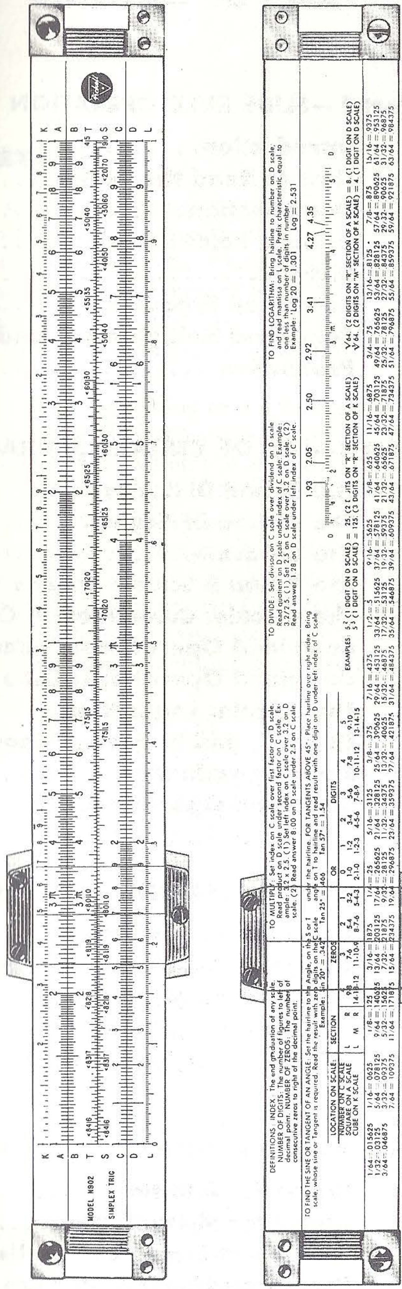
When people have difficulty in learning to use a slide rule, usually it is not because the instrument is difficult to use. The reason is likely to be that they do not understand the mathematics on which the instrument is based, or the formulas they are trying to evaluate. Some slide rule manuals contain relatively extensive explanations of the theory underlying the operations. In this manual such theory has deliberately been kept to a minimum. It is assumed that the *theory* of exponents, of logarithms, of trigonometry, and of the slide rule is known to the reader, or will be recalled or studied by reference to formal textbooks on these subjects. This is a brief manual on operational technique and is not intended to be a textbook or workbook.

Some of the special scales described in this manual may not be available on your slide rule. All of the illustrations and problems shown can be worked on the slide rule you purchased. However, the special scales simplify the calculations. Pickett Slide Rules are available with all of the special scales shown in this manual and also with Log Log features not described here.

TABLE OF CONTENTS



MODEL 1010 TRIG



MODEL 902 SIMPLEX TRIG

PART 1. SLIDE RULE OPERATION

INTRODUCTION

The slide rule is a simple tool for finding numerical answers to involved mathematical problems. To solve problems easily and with confidence it is necessary to have a clear understanding of the operation of your slide rule. Speed and accuracy will soon reward the user who makes a careful study of the scale arrangements and the manual.

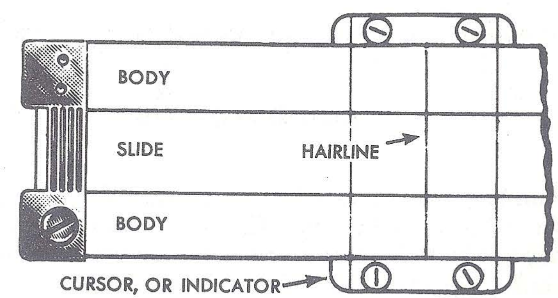
The table below shows some of the mathematical operations which can be done easily and quickly with an ordinary slide rule.

OPERATIONS	INVERSE OPERATIONS
Multiplying two or more numbers	Dividing one number by another
Squaring a number	Finding the square root of a number
Cubing a number	Finding the cube root of a number
Finding the logarithm of a number	Finding a number whose logarithm is known
Finding the sine, cosine, or tangent of an angle	Finding an angle whose sine, cosine, or tangent is known

Various *combinations* of these operations (such as multiplying two numbers and then finding the square root of the result) are also easily done.

The slide rule consists of three parts: (1) the body (upper and lower fixed bars); (2) the slide; (3) the cursor or indicator. The scales on the body and slide are arranged to work together in solving problems. The hairline on the indicator is used to help the eyes in reading the scales and in adjusting the slide.

Fig. 1



Each scale is named by a letter (A, B, C, D, L, S, T) or other symbol at the end.

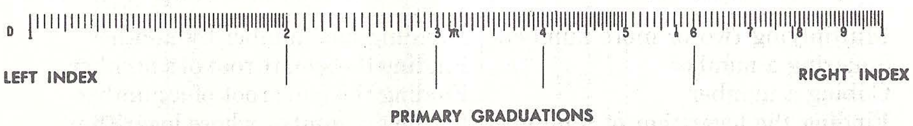
In order to use a slide rule, a computer must know: (1) how to read the scales; (2) how to "set" the slide and indicator for each operation to be done; and (3) how to determine the decimal point in the result.

HOW TO READ THE SCALES

The scale labeled C (on the slide) and the scale D (on the body) are used most frequently. These two scales are exactly alike. The total length of these scales has been separated into many smaller parts by fine lines called "graduations."

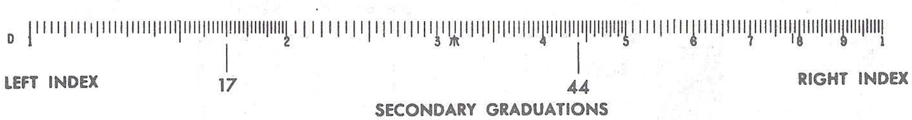
Some of these lines on the D scale have large numerals (**1**, **2**, **3**, etc.) printed just below them. These lines are called *primary* graduations. On the C scale the numerals are printed above the corresponding graduations. A line labeled **1** at the left end is called the *left* index. A line labeled **1** at the right end is called the *right* index.

Fig. 2



Next notice that the distance between **1** and **2** on the D scale has been separated into 10 parts by shorter graduation lines. These are the *secondary* graduations. (On 10 inch slide rules these lines are labeled with smaller numerals 1, 2, 3, etc. On 6 inch rules these lines are not labeled.) Each of the spaces between the larger numerals **2** and **3**, between **3** and **4**, and between the other primary graduations is also sub-divided into 10 parts. Numerals are not printed beside these smaller secondary graduations because it would crowd the numerals too much.

Fig. 3



When a number is to be located on the D scale, the *first* digit is located by use of the *primary* graduations. The *second* digit is located by use of the *secondary* graduations. Thus when the number 17 is located, the **1** at the left index represents the **1** in 17. The 7th secondary graduation represents the 7. When 44 is to be located, look first for primary graduation 4, and then for secondary graduation 4 in the space immediately to the right.

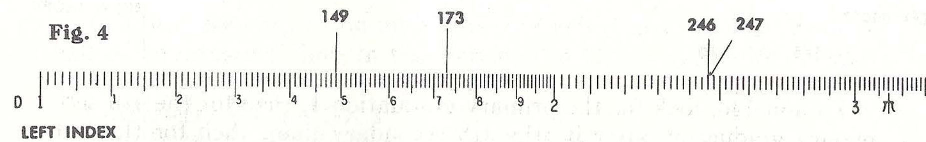
There are further sub-divisions, or *tertiary graduations*, on all slide rules. The meaning of these graduations is slightly different at different parts of the scale. It is also different on a 6 inch slide rule than on a 10 inch rule. For this reason a separate explanation must be given for each.

Tertiary graduations on 10 inch rules.

The space between each secondary graduation at the left end of the rule (over to primary graduation **2**) is separated into ten parts, but these shortest graduation marks are not numbered. In the middle part of the rule, between the primary graduations **2** and **4**, the smaller spaces between the *secondary* graduations are separated into five parts. Finally, the still smaller spaces between the secondary graduations at the right of **4** are separated into only two parts.

To find 173 on the D scale, look for primary division **1** (the left index). Then look for secondary division 7 (numbered). Then look for smaller subdivision 3, which is not numbered, but found as the 3rd very short graduation to the right of the longer graduation for 7.

Fig. 4

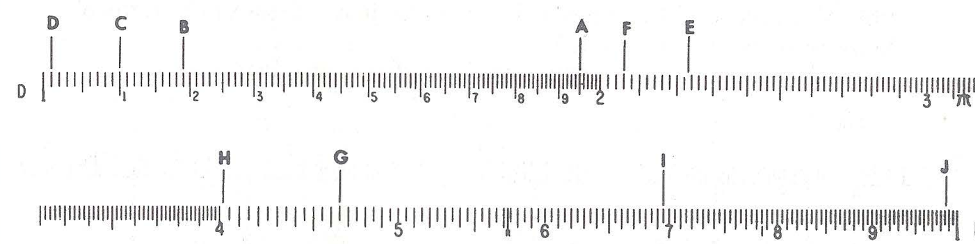


Similarly, 149 is found as the 9th small graduation mark to the right of the 4th secondary graduation mark to the right of primary graduation **1**.

To find 246, look for primary graduation **2**, then for the 4th secondary graduation after it (the 4th long line), then for the 3rd small graduation after it. The smallest spaces in this part of the scale are read as fifths. Since $\frac{3}{5} = \frac{6}{10}$, then the third graduation, marking *three fifths*, is at the same point as *six tenths* would be.

On the scale below are some sample readings for 10 inch scales.

Fig. 5



A: 195	F: 206
B: 119	G: 465
C: 110	H: 402
D: 101	I: 694
E: 223	J: 987

Tertiary graduations on 6 inch rules.

The space between each secondary graduation at the left end of the rule (over to primary graduation **2**) is separated into five parts. In the middle of the rule, between the primary graduations **2** and **5**, the smaller spaces between the secondary graduations are separated into two parts. Finally, the still smaller spaces between the secondary graduations at the right of **5** are not subdivided.

To find 170 on the D scale, look for the primary division **1** (the left index), then for the 7th secondary graduation.

Fig. 6

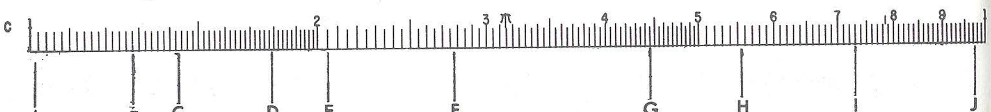
To find 146, look for the primary graduation **1**, then for the 4th secondary graduation after it (the 4th secondary line), then for the 3rd small graduation after it. The smallest spaces in this part of the scale are read as fifths. Since 3/5 = 6/10, then the third graduation, marking *three fifths*, is at the same point as *six tenths* would be.

The number 147 would be half of a small space beyond 146. With the aid of the hairline on the runner the position of this number can be located approximately by the eye. The small space is mentally "split" in half.

The number 385 is found by locating primary graduation **3** and then secondary graduation 8 (the 8th long graduation after 3). Following this, one observes that between secondary graduations 8 and 9 there is one short mark. Think of this as the "5 tenths" mark, which represents 385. The location of 383 can be found approximately by mentally "splitting" the space between 380 and 385 into fifths, and estimating where the 3rd "fifths" mark would be placed. It would be just a little to the right of halfway between 380 and 385.

On the scale below are some sample readings for 6 inch scales.

Fig. 7



A: 104	F: 276
B: 128	G: 445
C: 143	H: 555
D: 179	I: 731
E: 205	J: 970

The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, used in writing numbers are called *digits*. One way to describe a number is to tell how many digits are used in writing it. Thus 54 is a "two-digit number", and 1,348,256 is a "seven-digit number." In many computations only the first two or three digits of a number need to be used to get an approximate result which is accurate enough for practical purposes. Usually not more than the first three digits of a number can be "set" on a six inch slide rule scale. In many practical problems this degree of accuracy is sufficient. When greater accuracy is desired, a ten inch slide rule is generally used.

For an explanation of how these scales are constructed, see Page 58.

MULTIPLICATION

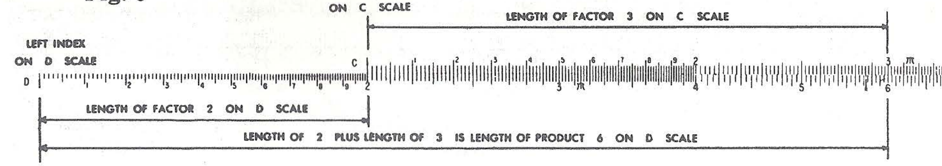
Numbers that are to be multiplied are called factors. The result is called the product. Thus, in the statement 6 × 7 = 42, the numbers 6 and 7 are factors, and 42 is the product.

- Example: Multiply 2 × 3.

Setting the Scales: Set the left index of the C scale on 2 of the D scale. Find 3 on the C scale, and below it read the product, 6 on the D scale.

Think: The length for 2 plus the length for 3 will be the length for the product. This length, measured by the D scale, is 6.

Fig. 8



- Example: Multiply 4 × 2.

Setting the Scales: Set the left index of the C scale on 4 of the D scale. Find 2 on the C scale, and below it read the product, 8, on the D scale.

Think: The length for 4 plus the length for 2 will be the length for the product. This length, measured by the D scale, is 8.

***Rule for Multiplication:* Over one of the factors on the D scale, set the index of the C scale. Locate the other factor on the C scale, and directly below it read the product on the D scale.**

This rule may be shown in the form of a diagram as follows.*

To find $P = x \cdot y$, set: $\frac{C}{D}\Big|\frac{1}{x}\Big|\frac{y}{P}$.

To find $P = 2 \times 3$, set: $\frac{C}{D}\Big|\frac{1}{2}\Big|\frac{3}{P}$.

*This device for showing slide rule settings has been used for many years. See, for example, Lipka, Joseph, *Graphical and Mechanical Computation.* John Wiley and Sons, Inc., (New York, 1918), p. 10.

• Example: Multiply 2.34 × 36.8.

Estimate the result: First note that the result will be roughly the same as 2 × 40, or 80; that is, there will be two digits to the left of the decimal point. Hence, we can ignore the decimal points for the present and multiply as though the problem was 234 × 368.

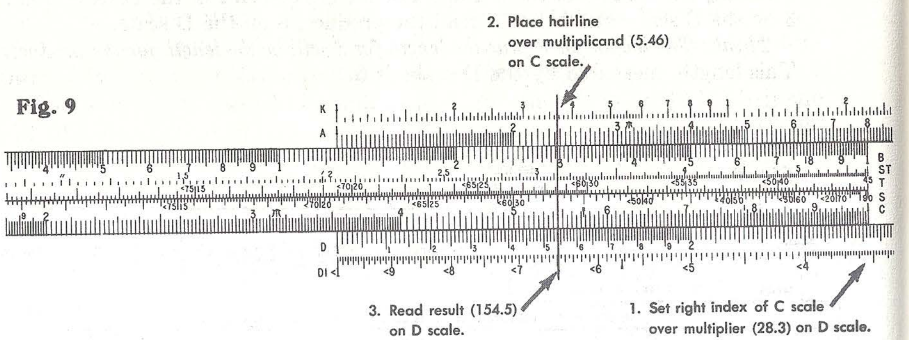
Set the Scales: Set the left index of the C scale on 234 of the D scale. Find 368 on the C scale and read product 861 on the D scale.

Think: The length for 234 plus the length for 368 will be the length for the product. This length is measured on the D scale. Since we already knew the result was somewhere near 80, the product must be 86.1, approximately.

• Example: Multiply 28.3 × 5.46.

Note first that the result will be about the same as 30 × 5, or 150. Note also that if the left index of the C scale is set over 283 on the D scale, and 546 is then found on the C scale, the slide projects so far to the right of the body that the D scale is no longer below the 546. When this happens, the other index of the C scale can be used. That is, set the *right* index on the C scale over 283 on the D scale. Find 546 on the C scale and below it read the product on the D scale. The product is 154.5.

Fig. 9



These examples illustrate how in simple problems the decimal point can be placed by use of an estimate. Other methods of placing the decimal point will be explained later in this manual.

In doing these simple examples it is not really necessary to use the hairline on the cursor. However, most experienced slide rule users do use it. Thus to multiply 2.34 × 36.8, they first place the hairline over 234 on D. Then they move the slide so that the index 1 of the C scale is under the hairline. Then they move the cursor to place the hairline over 368 of the C scale. The answer 86.1 is then read under the hairline on the D scale.

To prevent the slide from changing position while the hairline is being moved, pinch the upper and lower bars of the body together. They are slightly flexible and will hold the slide until the pressure is released.

Later sections of this manual will explain how to do multiplication by using other scales. For some examples the use of these other scales makes the work easier. Use the C and D scales for Problem Set I. (Answers are given in the back of this manual.)

PROBLEM SET I

1. 15 × 3.7	**2.** 4.2 × 19
3. 62.1 × 3.9	**4.** 9.54 × 16.7
5. 1.85 × π	**6.** 20.4 × 410
7. 9.3 × 8.7	**8.** 71.5 × 5.05
9. 53.6 × 2.3	**10.** 3.76 × 18.2
11. 298 × 0.26	**12.** 18.3 × 0.54
13. 0.81 × 92.2	**14.** 112 × 824
15. 0.29 × 0.31	**16.** 0.0215 × 3.79

DECIMAL POINT LOCATION

In the discussion which follows, it will occasionally be necessary to refer to the number of "digits" and number of "zeros" in some given numbers.

When numbers are greater than 1 the number of *digits* to the left of the decimal point will be counted. Thus 734.05 will be said to have 3 digits. Although as written the number indicates accuracy to *five* digits, only three of these are at the left of the decimal point.

Numbers that are less than 1 may be written as *decimal fractions.** Thus .673, or six-hundred-seventy-three thousandths, is a decimal fraction. Another example is .000465. In this number three zeros are written to show where the decimal point is located. One way to describe such a number is to tell how many zeros are written to the right of the decimal point before the first non-zero digit occurs.

In scientific work a zero is often written to the left of the decimal point, as in 0.00541. This shows that the number in the units' place is definitely 0, and that no digits have been carelessly omitted in writing or printing. The zeros will *not* be counted unless they are (a) at the *right* of the decimal point, (b) before or at the *left* of the first non-zero digit, and (c) are not between other digits. The number 0.000408 will be said to have 3 zeros (that is, the number of zeros between the decimal point and the 4).

Method I. In many, perhaps a majority, of the problems met in genuine applications of mathematics to practical affairs, the position of the decimal point in the result can be determined by what is sometimes called "common sense." There is usually only one place for the decimal point in which the answer is "reasonable" for the problem. Thus, if the calculated speed in miles per hour of a powerful new airplane comes out to be 4833, the decimal point clearly belongs between the 3's, since 48 m.p.h. is too small, and 4833 m.p.h. is too large for such a plane. In some cases, however, the data are such that the position of the point in the final result is not easy to get by inspection.

Method II. Another commonly used method of locating the decimal point is by estimation or approximation. For example, when the slide rule is used to find 133.4 × 12.4, the scale reading for the result is 1655, and

*Only positive real numbers are being considered in this discussion.

the decimal point is to be determined. By rounding off the factors to 133.0 × 10.0, one obtains 1330 by mental arithmetic. The result would be somewhat greater than this but certainly contains four digits on the left of the decimal point. The answer, therefore, must be 1655.

Method III. In scientific work numbers are often expressed in *standard form*. For example, 428 can be written 4.28×10^2, and 0.00395 can be written as 3.95×10^{-3}. When a number is written in standard form it always has two factors. The first factor has one digit (not a zero) on the left of the decimal point, and usually other digits on the right of the decimal point. The other factor is a power of 10 which places the decimal point in its true position if the indicated multiplication is carried out. In many types of problems this method of writing numbers simplifies the calculation and the location of the decimal point.

When a number is written in standard form, the exponent of 10 may be called "the characteristic." It is the characteristic of the logarithm of the number to base 10. The characteristic may be either a positive or a negative number. Although the rule below appears long, in actual practice it may be used with great ease.

***Rule.* To express a number in standard form:**

(a) place a decimal point at the right of the first non-zero digit.*

(b) start at the right of the first non-zero digit in the original number and count the digits and zeros passed over in reaching the decimal point. The result of the count is the numerical value of the characteristic, or exponent of 10. If the original decimal point is toward the right, the characteristic is *positive* (+). If the original decimal point is toward the left, the characteristic is *negative* (−). Indicate that the result of (a) is to be multiplied by 10 with the exponent thus determined in (b).

• EXAMPLES:

Number	Number in standard form.	Characteristic
(a) 5,790,000	5.79×10^6	6
(b) 0.000283	2.83×10^{-4}	−4
(c) 44	4.4×10^1	1
(d) 0.623	6.23×10^{-1}	−1
(e) 8.15	8.15×10^0	0
(f) 461,328	4.61328×10^5	5
(g) 0.0000005371	5.371×10^{-7}	−7
(h) 0.0306	3.06×10^{-2}	−2
(i) 80.07	8.007×10^1	1

If a number given in standard form is to be written in "ordinary" form, the digits should be copied, and then starting at the right of the first digit the number of places indicated by the exponent should be counted, supplying zeros as necessary, and the point put down. If the exponent is positive, the count is toward the right; if negative, the count

is toward the left. This converse application of the rule may be verified by studying the examples given above.

Consider now the calculation of 5,790,000 × 0.000283. From examples (a) and (b) above, this can be written $5.79 \times 10^6 \times 2.83 \times 10^{-4}$, or by changing order and combining the exponents of 10, as $5.79 \times 2.83 \times 10^2$. Then since 5.79 is near 6, and 2.83 is near 3, the product of these two factors is known to be near 18. The multiplication by use of the C and D scales shows it to be about 16.39, or 1.639×10^1. Hence, $5.79 \times 2.83 \times 10^2 = 1.639 \times 10^1 \times 10^2 = 1.639 \times 10^3 = 1639$. If, however, one has 5,790,000 ÷ 0.000283, the use of standard form yields

$$\frac{5.79 \times 10^6}{2.83 \times 10^{-4}} = 2.04 \times 10^{6-(-4)} = 2.04 \times 10.^{10}$$

In scientific work the result would be left in this form, but for popular consumption it would be written as 20,400,000,000. The general rule is as follows.

***Rule.* To determine the decimal point, first express the numbers in standard form. Carry out the indicated operations of multiplication or division, using the laws of exponents to combine the exponents until a single power of 10 is indicated. If desired, write out the resulting number, using the final exponent of 10 to determine how far, and in what direction, the decimal point in the coefficient should be moved.**

The theory of exponents and the rules of operation with signed numbers are both involved in a complete treatment of this topic. In this manual it is assumed that the reader is familiar with this theory. The "standard form" method of placing the decimal point can be used with all types of calculations. It is the most general, scientific, and reliable method, but for many problems it requires skill in the use of algebra.

Method IV. Special scales, called *Decimal-Keeper* Scales, have been devised to help in locating the decimal point. These scales are found only on *Decimal-Keeper* slide rules. The use of these scales is explained on pages 55 to 56 of this manual.

There are some other methods, but they are difficult to use and remember. This is especially true whenever the problem is more complicated than a simple multiplication or division of two numbers. For these reasons these methods will not be explained here.

• EXAMPLES:

(a) 34,900 × 0.0012. Use Method II, and think of this as about

$$30{,}000 \times \frac{12}{10{,}000},$$

which is 36. Use the C and D scales to multiply 349 by 12, obtaining 419. Since the answer must be near 36, it must be 41.9.

(b) 34,900 × 0.0012. If Method III is used, write the numbers in standard form. Thus $34{,}900 = 3.49 \times 10^4$ and $0.0012 = 1.2 \times 10^{-3}$.

*In using this rule, "first" is to be counted from the left; thus, in 3246, the digit 3 is "first."

Then

$$3.49 \times 10^4 \times 1.2 \times 10^{-3} = 3.49 \times 1.2 \times 10^1$$
$$= 4.19 \times 10^1$$
$$= 41.9$$

(c) 6,230,000 × 0.00048. If Method II is used, think of this as about

$$6{,}000{,}000 \times \frac{50}{100{,}000},$$

which by "cancelling" zeros is easily found to be 3000. Use the C and D scales to multiply 623 × 48, obtaining 299. Since the answer is near 3000, it must be 2990.

(d) 6,230,000 × 0.00048. If Method III is used, write the numbers in standard form.

$$6.23 \times 10^6 \times 4.8 \times 10^{-4} = 6.23 \times 4.8 \times 10^2$$
$$= 29.9 \times 10^2$$
$$= 2.99 \times 10^3$$
$$= 2990.$$

PROBLEM SET II

1. 24.9 × 13.1	**2.** 37.2 × 16.6
3. 1728 × 367	**4.** 1836 × 298
5. 0.0232 × 469	**6.** 0.0145 × 543
7. 0.0715 × 3.48	**8.** 0.0627 × 4.34
9. 0.43 × 0.0027	**10.** 0.38 × 0.0042
11. 3,240 × 0.00039	**12.** 2,650 × 0.00047
13. 0.0061 × π	**14.** 0.0072 × π
15. 0.000,53 × 0.00623	**16.** 0.000,478 × 0.0069

DIVISION

In mathematics, division is the opposite or *inverse* operation of multiplication. In using a slide rule this means that the process for multiplication is reversed. To help in understanding this statement, set the rule to multiply 2 × 4 (see page 7). Notice the result 8 is found on the D scale under 4 of the C scale. Now to divide 8 by 4 these steps are reversed. First find 8 on the D scale, set 4 on the C scale over it, and read the result 2 on the D scale under the index of the C scale.

Think: From the length for 8 (on the D scale) *subtract* the length for 4 (on the C scale). The length for the difference, read on the D scale, is the result, or quotient.

With this same setting you can read the quotient of 6 ÷ 3, or 9 ÷ 4.5, and in fact all divisions of one number by another in which the result is 2.

***Rule for Division:* Set the *divisor* (on the C scale) opposite the number to be divided (on the D scale). Read the result, or quotient, on the D scale under the index of the C scale.**

This may be shown in a diagram as follows. To find $Q = x \div y$, set

$$\begin{array}{c|c|c} C & y & 1 \\ \hline D & x & Q \end{array}.$$

• EXAMPLES:

(a) Find 63.4 ÷ 3.29. The quotient must be near 20, since 60 ÷ 3 = 20. Set indicator on 63.4 of the D scale. Move the slide until 3.29 of the C scale is under the hairline. Read the result 19.27 on the D scale at the C index.

(b) Find 26.4 ÷ 47.7. Since 26.4 is near 25, and 47.7 is near 50, the quotient must be roughly 25/50 = ½ = 0.5. Set 47.7 of C opposite 26.4 of D, using the indicator to aid the eyes. Read 0.553 on the D scale at the C index.

(c) Find 371 ÷ 0.00352. Write the problem in the form

$$\frac{3.71 \times 10^2}{3.52 \times 10^{-3}} = \frac{3.71}{3.52} \times 10^5.$$

Set the hairline over 371 on D. Move the slide so 352 on C is under the hairline. At the index of C read 1054 on D. Start at the right of 1 and count 5 places, supplying zeros as needed, to obtain the result: 105,400.

PROBLEM SET III

1. 47 ÷ 29	**2.** 83 ÷ 7
3. 75 ÷ 92	**4.** 69 ÷ 79
5. 137 ÷ 513	**6.** 152 ÷ 567
7. 0.049 ÷ 0.023	**8.** 17.3 ÷ 231
9. 924 ÷ 26.3	**10.** 847 ÷ 31.6
11. 0.0763 ÷ 8.7	**12.** 0.0822 ÷ 7.6
13. 428 ÷ 0.34	**14.** 398 ÷ 0.26
15. 564 ÷ 0.047	**16.** 621 ÷ 0.039
17. 8570 ÷ 0.0219	**18.** 7630 ÷ 0.0198
19. 0.00146 ÷ 32.8	**20.** 0.00231 ÷ 41.3

CONTINUED PRODUCTS

Sometimes the product of three or more numbers must be found. These "continued" products are easy to get on the slide rule. Below is a *general rule* for continued products: $a \times b \times c \times d \times e \cdots$

Set hairline of indicator at a on D scale.
Move index of C scale under hairline.
Move hairline over b on the C scale.
Move index of C scale under hairline.
Move hairline over c on the C scale.
Move index of C scale under hairline.
Continue moving hairline and index alternately until all numbers have been set.
Read result under the hairline on the D scale.

• EXAMPLE: Multiply 38.2 × 1.65 × 8.9.

Estimate the result as follows: 40 × 1 × 10 = 400. The result should be, very roughly, 400.

Setting the Scales: Set left index of the C scale over 382 on the D scale. Find 165 on the C scale, and *set the hairline* on the indicator on it. The product of 382 × 165 could now be read under the hairline on the D scale, but this is not necessary. Move the index on the slide under the hairline. In this example if the *left* index is placed under the hairline, then 89 on the C scale falls outside the D scale. Therefore move the *right* index under the hairline. Move the hairline to 89 on the C scale and read the result (561) under it on the D scale.

PROBLEM SET IV

1. $2.9 \times 3.4 \times 7.5$
2. $17.3 \times 43 \times 9.2$
3. $343 \times 91.5 \times 0.00532$
4. $19 \times 407 \times 0.0021$
5. $13.5 \times 709 \times 0.567 \times 0.97$

COMBINED MULTIPLICATION AND DIVISION

Many problems call for both multiplication and division.

• Examples:

(a) $\dfrac{42 \times 37}{65}$

First, set the division of 42 by 65; that is, set 65 on the C scale opposite 42 on the D scale. The quotient, .646, need not be read. Move the hairline on indicator to 37 on the C scale. Read the result 239 on the D scale under the hairline. Since the fraction $\frac{42}{65}$ is about equal to $\frac{2}{3}$, the result is about two-thirds of 37, or 23.9.

(b) $\dfrac{273 \times 548}{692 \times 344}$.

Set 692 on the C scale opposite 273 on the D scale. Move the hairline to 548 on the C scale. Move the slide to set 344 on the C scale under the hairline. Read the result .628 on the D scale under the C index.

In general, to do computations of the type $\dfrac{a \times c \times e \times g \cdots}{b \times d \times f \times h \cdots}$, set the rule to divide the first factor in the numerator a by the first factor in the denominator b, move the hairline to the next factor in the numerator c; move the slide to set next factor in denominator, d, under the hairline. Continue moving hairline and slide alternately for other factors (e, f, g, h, *etc.*). Read the result on the D scale. If there is one more factor in the numerator than in the denominator, the result is under the hairline. If the number of factors in numerator and denominator is the same, the result is under the C index. Sometimes the slide must be moved so that one index replaces the other. This statement assumes that up to this point only the C and D scales are being used. Later sections will describe how this operation may be avoided by the use of other scales.

• Example:

(a) $\dfrac{2.2 \times 2.4}{8.4}$

If the rule is set to divide 2.2 by 8.4, the hairline cannot be set over 2.4 of the C scale and at the same time remain on the rule. Therefore the hairline is moved to the C index (opposite 262 on the D scale) and the slide is moved end for end to the right (so that the *left* index falls under the hairline and

over 262 on the D scale). Then the hairline is moved to 2.4 on the C scale and the result .63 is read on the D scale.

If the number of factors in the numerator exceeds the number in the denominator by more than one, the numbers may be grouped, as shown below. After the value of the *group* is worked out, it may be multiplied by the other factors in the usual manner.

$$\left(\frac{a \times b \times c}{m \times n}\right) \times d \times e$$

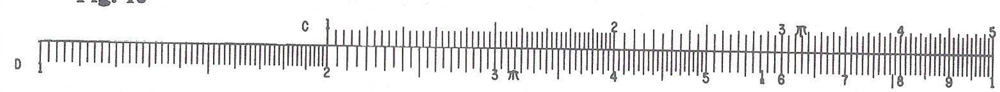
PROBLEM SET V

1. $\dfrac{27 \times 43}{19}$
2. $\dfrac{5.17 \times 1.25 \times 9.33}{4.3 \times 6.77}$
3. $\dfrac{842 \times 2.41 \times 173}{567 \times 11.52}$
4. $\dfrac{1590 \times 3.64 \times 0.763}{4.39 \times 930}$
5. $\dfrac{0.0237 \times 3970 \times 32 \times 6.28}{0.00029 \times 186000}$
6. $\dfrac{231 \times 58.6 \times 4930}{182.5 \times 3770}$
7. $\dfrac{875 \times 1414 \times 2.01}{661 \times 35.9}$
8. $\dfrac{558 \times 1145 \times 633 \times 809}{417 \times 757 \times 354}$
9. $\dfrac{0.691 \times 34.7 \times 0.0561}{91{,}500}$
10. $\dfrac{19.45 \times 7.86 \times 361 \times 64.4}{32.6 \times 9.74}$

PROPORTION

Problems in proportion are very easy to solve. First notice that when the index of the C scale is opposite 2 on the D scale, the ratio 1 : 2 or $\frac{1}{2}$ is at the *same time* set for all other opposite graduations; that is, 2 : 4, or 3 : 6, or 2.5 : 5, or 3.2 : 6.4, etc. It is true in general that for any setting of the slide, many equal ratios are shown.

Fig. 10



Suppose one of the terms of a proportion is unknown. The proportion can be written as $\frac{a}{b} = \frac{c}{x}$, where a, b, and c, are known numbers and x is to be found.

***Rule:* Set a on the C scale opposite b on the D scale. Under c on the C scale read x on the D scale.**

This may be shown in a diagram as follows: $\begin{array}{c|c|c} C & a & c \\ \hline D & b & x \end{array}$.

• EXAMPLE:

Find x if $\frac{3}{4} = \frac{5}{x}$. Set 3 on C opposite 4 on D. Under 5 on C read 6.67 on D.

The proportion above could also be written $\frac{b}{a} = \frac{x}{c}$, or "inverted," and exactly the same rule could be used. Morever, if C and D are interchanged in the above rule, it will still hold if "under" is replaced by "over." It then reads as follows:

Set a on the D scale opposite b on the C scale. Over c on the D scale read x on the C scale.

***Rule:* In solving proportions, keep in mind that the position of the numbers as set on the scales is the same as it is in the proportion written in the form $\frac{a}{b} = \frac{c}{d}$.**

Since one setting of the slide produces many equal ratios, it is possible to solve "continued" proportions by moving only the hairline.

• EXAMPLE:

Find a, f, and g in the following.

$$\frac{a}{12.5} = \frac{2.4}{3.0} = \frac{32}{f} = \frac{g}{7.5}$$

Set 2.4 on C over 3.0 on D. Move the hairline:
over 12.5 on D, read $a = 10$ on C;
over 32 on C, read $f = 40$ on D;
over 7.5 on D, read $g = 6$ on C.

Sometimes the slide must be moved "end-for-end" because a reading falls outside the body of the rule. Decimal points in continued proportions are usually easy to locate by estimation (i. e., Method II).

Proportions can also be solved *algebraically*. Then $\frac{a}{b} = \frac{c}{x}$ becomes $x = \frac{bc}{a}$, and this may be computed as combined multiplication and division.

In practical work it is often necessary to change a set of numbers (like those in the table below) all to per cent.

• EXAMPLE: Find the missing per cents in the table.

Name	Number	Per cent
Able	17.6	12.4
Baker	21.4	15.1
Charlie	28.2	—
Dog	30.6	—
Fox	44.2	—
Total	142.0	100.0

To figure the per cents, each number (17.6, 21.4, etc.) must be divided by the "base" or total, 142. Instead of re-setting the slide for each division, it is easier to divide 1 by 142, then multiply the result by 17.6, by 21.4, etc. Note that moving the hairline over 17.6 accomplishes the multiplication. This method replaces five divisions by one division and five multiplications. Only one setting of the slide is needed.

This problem can be viewed as a set of proportions, namely:

$$\frac{17.6}{142} = \frac{a}{100}; \frac{21.4}{142} = \frac{b}{100}; \frac{28.2}{142} = \frac{c}{100}; \text{ etc.}$$

These can be written in the form

$$\frac{100}{142} = \frac{a}{17.6} = \frac{b}{21.4} = \frac{c}{28.2} = \frac{d}{30.6} = \frac{e}{44.2}.$$

Set 142 of Cover 1 of D. Move-hairline over 17.6 on C. Read 12.4% on D. Then move hairline over 21.4 of C. Read 15.1% on D. Keep moving hairline successively over other numbers in the table, reading per cents on D.

Finish the table, and check by adding the per cents. The sum should be 100%.

In other problems that require repeated divisions by the same number, a similar procedure may be followed.

PROBLEM SET VI

1. $\frac{8}{5} = \frac{2}{a}$
2. $\frac{14}{17} = \frac{35}{a}$
3. $\frac{12}{7} = \frac{23}{x}$
4. $\frac{18}{91} = \frac{13}{x}$
5. $\frac{y}{42.5} = \frac{13.2}{1.87}$
6. $\frac{90.5}{m} = \frac{3.42}{1.54}$
7. $\frac{43.6}{89.2} = \frac{x}{2550}$
8. $\frac{34.5}{73.8} = \frac{c}{2430}$
9. $\frac{0.063}{0.51} = \frac{34.1}{d}$
10. $\frac{0.0237}{0.0542} = \frac{x}{60.4}$
11. $\frac{16}{19} = \frac{a}{27} = \frac{31}{b}$
12. $\frac{x}{23.8} = \frac{18.7}{19.2} = \frac{y}{34.5}$
13. $\frac{4.8}{a} = \frac{50}{37} = \frac{112}{b}$
14. $\frac{r}{16.0} = \frac{64}{s} = \frac{2.7}{0.32}$
15. $\frac{8.23}{76.1} = \frac{x}{4.02} = \frac{11.1}{y}$
16. $\frac{1}{3} = \frac{a}{4.8} = \frac{240}{b} = \frac{c}{12.7}$

MIXED PROBLEMS, SET VII

1. 143×0.387
2. 168×0.324
3. $18.9 \times 132 \times 0.0481$
4. $22.9 \times 116 \times 0.524$
5. $832 \div 6.41$
6. $716 \div 8.32$
7. $\dfrac{643 \times 8.12}{5.19}$
8. $\dfrac{469 \times 757}{5.13}$
9. $\dfrac{7}{9} = \dfrac{3.2}{x}$
10. $\dfrac{5}{11} = \dfrac{6.8}{x}$
11. $1 \div 3.43$
12. $1 \div 2.78$
13. 29.8×4.87
14. 68.3×2.91
15. $79.1 \times 3.62 \times 5.55$
16. $93.2 \times 22.1 \times 0.625$
17. $16.35 \div 8.02$
18. $14.62 \div 7.03$
19. $\dfrac{11.95 \times 9.12}{3.40}$
20. $\dfrac{16.28 \times 5.37}{4.60}$
21. $\dfrac{2.81}{6.02} = \dfrac{x}{8.11}$
22. $\dfrac{3.74}{7.08} = \dfrac{x}{8.81}$
23. $.0642 \times 80.6$
24. 0.0824×60.3
25. $0.0427 \times 91.4 \times 169$
26. $0.0231 \times 82.2 \times 182$
27. $40.7 \div 13.3$
28. $61.3 \div 15.1$
29. $\dfrac{7.75 \times .0414}{1.91}$
30. $\dfrac{8.95 \times 0.314}{2.09}$

PRACTICAL PROBLEMS, SET VIII

1. A stock that cost $27.60 a share paid a dividend of $2.25. What rate of return (per cent) was this?
2. At 7½¢ a square foot, how much will it cost to cover an area of 860 square feet?
3. Find the cost of carpet to cover a floor 13′ × 15′ at $9.85 per square yard.

   $$\frac{13 \times 15 \times 9.85}{9} = ? \text{ or } 4.33 \times 5 \times 9.85 = ?$$
4. A jet plane flew at the rate of 631 mi. per hr. Change this to feet per second.

   $$\frac{631 \times 5280}{3600} = ?$$
5. It required 3½ days to dig a ditch 520 feet long. At the same rate how many days should be required to dig the next 370 feet?
6. In the fifth annual Pan American Road Race one car travelled the 1910 mile course in a little less than 17.7 hours. What was the average speed?
7. If the voltage is 110 and the resistance is 73 ohms, what should the current be?

   $$I = \frac{E}{R}, \text{ or } I = \frac{110}{73}.$$

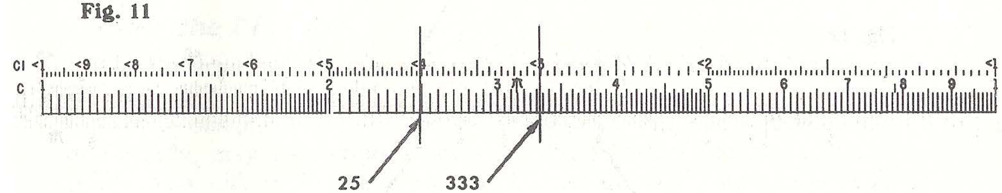
PART 2. USE OF CERTAIN SPECIAL SCALES

THE CI AND DI SCALES

It should be understood that the use of the CI and DI scales does not increase the power of the instrument to solve problems. In the hands of an experienced computer, however, these scales are used to reduce the number of settings or to avoid the awkwardness of certain settings. In this way the speed can be increased and errors minimized.

The CI scale on the slide is just like the C scale except that it *increases from right to left.* Starting at the right end, and reading toward the left, one finds 2 of CI just above 5 of C, the 3 of CI is above 333 of C, the 4 of CI is above 25 of C, etc. A small arrowhead (<) is placed at the left of the numeral to show that the scale values increase toward the left. The DI scale below the D scale is similar to the CI scale.

Fig. 11



Reciprocals. The reciprocal of a number a is $1 \div a$ or $\frac{1}{a}$. The C and CI scales (or the D and DI scales) may be used for finding reciprocals.

***Rule.* When any number is set under the hairline on the C scale its reciprocal is found under the hairline on the CI scale, and conversely.**

• Examples:
(a) Find 1/2.4. Set 2.4 on C. Read .417 directly above on CI.
(b) Find 1/60.5. Set 60.5 on C. Read .0165 directly above on CI. Or, set 60.5 on CI, read .0165 directly below on C.
(c) Find $1/\pi$. Place the hairline over π on D. Read 0.318 directly below on DI.

Multiplication. When the C and D scales are used for multiplication, it is sometimes necessary to change the slide "end-for-end." This happens if the factor on C is beyond the D scale. In division this never happens and the answer can always be read. The CI scale can be used to change a multiplication to an equivalent division. When multiplication is done this way, the slide never has to be changed end-for-end. Note that

$$P \doteq a \times b = a \div \left(\frac{1}{b}\right) = b \div \left(\frac{1}{a}\right).$$

***Rule:* The product of any two factors is equal to the one of the factors divided by the reciprocal of the other factor.**

This may be shown in diagrams as follows.

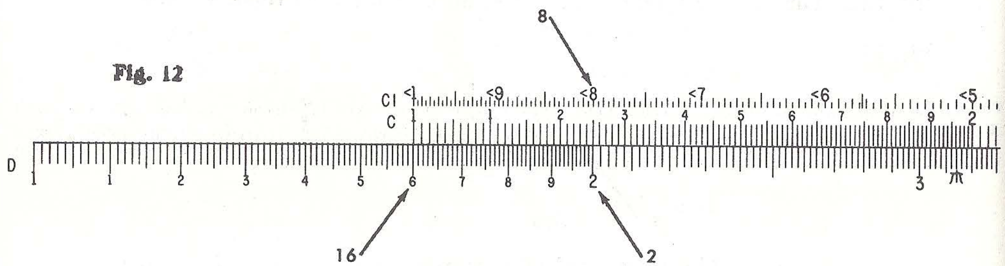
$$\begin{array}{c|c|c} \text{CI} & b & 1 \\ \hline \text{D} & a & \text{P} \end{array} \quad \text{or} \quad \begin{array}{c|c|c} \text{CI} & a & 1 \\ \hline \text{D} & b & \text{P} \end{array}$$

- EXAMPLES:

(a) Find P = 2 × 8. Suppose 1 of C is placed over 2 on D. Then 8 on C is beyond the D scale. Hence *divide* 2 by $\frac{1}{8}$. To do this set 8 on CI over 2 on D. Under the index of CI read P = 16 on D.

$$\begin{array}{c|c|c} \text{CI} & 8 & 1 \\ \hline \text{D} & 2 & \text{P} \end{array}$$

Fig. 12



(b) Find 13.6 × 82. Set the hairline over 13.6 on D. Move the slide so 82 of CI is under the hairline. At the index of CI read 1115.

Division. The CI scale is useful in replacing a division by a multiplication. Since $\frac{a}{b} = a \times (1/b)$, any division may be done by multiplying the numerator (or dividend) by the reciprocal of the denominator (or divisor). This process may often be used to avoid settings in which the slide projects far outside the rule. For example, consider Q = 16 ÷ 8. If 8 on C is set over 16 on D, the slide extends far out to the left. This example may be done more easily by thinking of it as Q = 16 × (1/8). Then the setting can be made as follows (see Fig. 12.):

$$\begin{array}{c|c|c} \text{CI} & 1 & 8 \\ \hline \text{D} & 16 & 2 \end{array}$$

- EXAMPLES:

(a) Find 13.6 ÷ 87.5. Consider this as 13.6 × 1/87.5. Set left index of the C scale on 13.6 of the D scale. Move hairline to 87.5 on the CI scale. Read the result, .155, on the D scale.

(b) Find 72.4 ÷ 1.15. Consider this as 72.4 × 1/1.15. Set right index of the C scale on 72.4 of the D scale. Move hairline to 1.15 on the CI scale. Read 63.0 under the hairline on the D scale.

Combined multiplication and division

By use of the CI scale, factors may be transferred from the numerator to the denominator of a fraction (or vice-versa) in order to make the rettings more readily.

- EXAMPLES:

(a) Find $\frac{13.6}{4.13 \times 2.79}$. This is the same as $\frac{13.6 \times (1/2.79)}{4.13}$.

Set 4.13 on the C scale opposite 13.6 on the D scale. Move hairline to 2.79 on the CI scale, and read the result, 1.180, on the D scale.

(b) Find 2.9 × 3.4 × 7.5. This may be written

$$\frac{2.9 \times 7.5}{(1/3.4)}.$$

Set the hairline over 2.9 on D. Move the slide so that 3.4 on CI is under the hairline. Then move the hairline to 7.5 of C. Read 73.9 on D under the hairline.

Using the DI scale

The DI scale has several important uses, of which the following is representative. Other uses will be explained later.

Expressions of the type 1/X, where X is some complicated expression or formula, may be computed by first finding the value of X. If the result for X falls on D, then 1/X may be read under the hairline on DI.

- EXAMPLE:

Find $\frac{1}{0.265 \times 138}$. Multiply 0.265 × 138 using the C and D scales. Read the reciprocal .0273 under the hairline on the DI scale. Or set the hairline on 265 of the DI scale, pull 138 of the C scale under the hairline, and read the result on the D scale under the left index of the C scale. This is equivalent to writing the expression as $\frac{(1/.265)}{138}$.

PROBLEM SET IX

Use the CI or DI scale in each of these problems.

1. $\frac{1}{7}$

2. $\frac{1}{35.2}$

3. 1/0.1795

4. 1 ÷ 6430

5. 1/0.024

6. 1/0.00417

7. 19.2 × 6.32

8. 23.1 × 6.8

9. 0.73 × 18.3

10. 0.84 × 11.5

11. 21.2 ÷ 9.5

12. 12.8 ÷ 8.2

13. 87.2 ÷ 2.04

14. 9.89 ÷ 3.13

15. $\frac{62.5}{32.1 \times 4.28}$

16. $\frac{463}{18.1 \times 67.5}$

17. 17.3 × 43 × 9.2

18. 19 × 407 × 0.0021

19. $\frac{1}{2.73 \times 0.56}$

20. $\frac{1}{37.5 \times 0.025}$

THE CF AND DF SCALES

It should be understood that the use of the CF and DF scales does not increase the power of the instrument to solve problems. In the hands of an experienced computer, however, these scales are used to reduce the number of settings. In this way the speed can be increased and errors minimized. When π on the C scale is opposite the right index of the D scale, about half the slide projects beyond the rule. If this part were cut off and used to fill in the opening at the left end, the result would be a "folded" C scale, or CF scale. Such a scale is printed at the top of the slide. It begins at π and the index is near the middle of the rule. The DF scale is similarly placed. Any setting of C on D is automatically set on CF and DF. Thus if 3 on C is opposite 2 on D, then 3 on CF is also opposite 2 on DF. The CF and DF scales can be used for multiplication and division in exactly the same way as the C and D scales.

The most important use of the CF and DF scales is to avoid resetting the slide. If a setting of the indicator cannot be made on the C or D scale, it can be made on the CF or DF scale.

- EXAMPLES:

(a) Find 19.2×6.38. Set left index of C on 19.2 of D. Note that 6.38 on C falls outside the D scale. Hence, move the indicator to 6.38 on the CF scale, and read the result 122.5 on the DF scale. Or set the index of CF on 19.2 of DF. Move indicator to 6.38 on CF and read 122.5 on DF.

(b) Find $\frac{8.39 \times 9.65}{5.72}$. Set 5.72 on C opposite 8.39 on D. The indicator cannot be moved to 9.65 of C, but it can be moved to this setting on CF and the result, 14.15, read on DF. Or the entire calculation may be done on the CF and DF scales.

These scales are also helpful in calculations involving π and $1/\pi$. When the indicator is set on any number N on D, the reading on DF is Nπ. This can be symbolized as (DF) $= \pi$(D). Then (D) $= \frac{\text{(DF)}}{\pi}$. This leads to the following simple rule.

Rule: **If the diameter of a circle is set on D, the circumference may be read immediately on DF, and conversely.**

- EXAMPLES:

(a) Find 5.6π. Set indicator over 5.6 on D. Read 17.6 under hairline on DF.

(b) Find $8/\pi$. Set indicator over 8 on DF. Read 2.55 under hairline on D.

(c) Find the circumference of a circle whose diameter is 7.2. Set indicator on 7.2 of D. Read 22.6 on DF.

(d) Find the diameter of a circle whose circumference is 121. Set indicator on 121 of DF. Read 38.5 on D.

Finally, these scales are useful in changing radians to degrees and conversely. Since π radians $=$ 180 degrees, the relationship may be written as a proportion $\frac{r}{d} = \frac{\pi}{180}$, or $\frac{r}{\pi} = \frac{d}{180}$.

Rule: **Set 180 of C opposite π on D. To convert radians to degrees, move indicator to *r* (the number of radians) on DF, read *d* (the number of degrees) on CF; to convert degrees to radians, move indicator to *d* on CF, read *r* on DF.**

There are also other convenient settings as suggested by the proportion. Thus one can set the ratio $\pi/180$ on the CF and DF scales and find the result from the C and D scales.

- EXAMPLES:

(a) The numbers 1, 2, and 7.64 are the measures of three angles in radians. Convert to degrees. Set 180 of C on π of D. Move indicator to 1 on DF, read 57.3° on CF. Move indicator over 2 of DF, read 114.6°. Move indicator to 7.64 of DF. Read 437° on CF.

(b) Convert 36° and 83.2° to radians. Use the same setting as in (a) above. Locate 36 on CF. Read 0.628 radians on DF. Locate 83.2 on CF. Read 1.45 radians on DF.

PROBLEM SET X

(Use CF or DF scale in each of these problems.)

1. 1.414×7.79
2. 2.14×57.6
3. $\frac{84.5 \times 7.59}{36.8}$
4. $2.65 \times \pi$
5. $13.2 \div \pi$
6. $\frac{0.1955 \times 23.7}{50.7 \times \pi}$
7. Find the circumference of a circle whose diameter is 84.7.
8. Find the radius of a circle whose circumference is 62.3.

THE CIF SCALE

Like the other special scales the CIF scale does not increase the power of the instrument to solve problems. It is used to reduce the number of settings or to avoid the awkwardness of certain settings. In this way the speed can be increased and errors minimized.

The CIF scale is a folded CI scale. Its relationship to the CF and DF scales is the same as the relation of the CI scale to the C and D scales.

- EXAMPLES:

(a) Find $2.07 \times 8.4 \times 16.1$. Set 1 of C over 2.07 on D. Notice that 8.4 on C is beyond the body of the rule, so move the indicator to 8.4 on CF. To multiply this by 16.1, move the slide so that 16.1 on CIF is under the hairline. Read the result, 280, on D at the index of C.

The multiplication by 16.1 could have been done by moving the slide so 1 of CF was under the hairline, and then moving the hairline to 16.1 on CF. In this case the answer is read under the hairline on DF. Notice this method requires an extra setting of the indicator which is saved by using CIF.

(b) Find $1/0.0613\pi$. Set the hairline over 613 on C. Read the result, 5.19, on CIF. Notice that this answer is obtained without any setting of the *slide*, and a single setting of the hairline.

(c) Find $(1/3\pi) \times 27$. Set the hairline over 27 on DF. Move the slide so 3 on C is under hairline. At 1 of C read result, 2.865 on D.

PROBLEM SET XI

(Use CIF to solve these problems.)

1. $1/0.43\pi$ **2.** $1/2\pi$

3. $\frac{1}{2\pi} \times 18.3$ **4.** $\frac{1}{4\pi} \times 0.082$

5. $\frac{1.37 \times 9.2}{\pi}$ **6.** $\frac{0.0237 \times 63.8}{\pi}$

THE A AND B SCALES: SQUARE ROOTS AND SQUARES

When a number is multiplied by itself the result is called the *square* of the number. Thus 25 or 5×5 is the square of 5. The factor 5 is called the *square root* of 25. Similarly, since $12.25 = 3.5 \times 3.5$, the number 12.25 is called the square of 3.5; also 3.5 is called the square root of 12.25. Squares and square roots are easily found on a slide rule.

Square Roots: To find square roots the A and D scales or the B and C scales are used.

***Rule:* The square root of any number located on the A scale is found below it on the D scale;**

$$\frac{A}{D}\left|\frac{n}{\sqrt{n}}\right|, \text{ or } \frac{B}{C}\left|\frac{n}{\sqrt{n}}\right|.$$

Also, the square root of any number located on the B scale (on the slide) is found on the C scale.

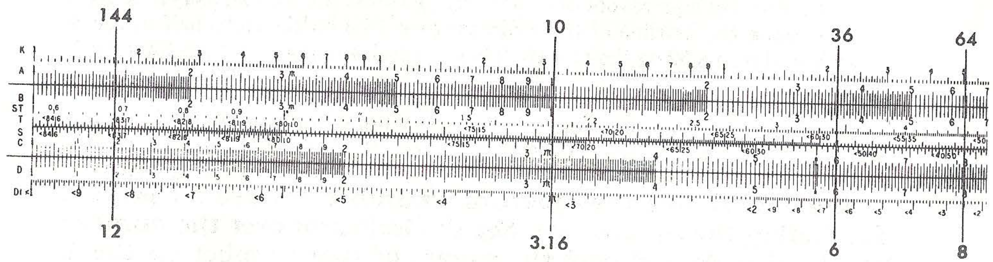
• EXAMPLE:

Find the $\sqrt{4}$. Place the hairline of the indicator over 4 on the left end of the A scale. The square root, 2, is read below on the D scale. Similarly the square root of 9 (or $\sqrt{9}$) is 3, found on the D scale below the 9 on the left end of the A scale.

Reading the Scales: The A scale is a contraction of the D scale itself. The D scale has been shrunk to half its former length and printed twice on the same line. To find the square root of a number between 0 and 10 the left half of the A scale is used (as in the examples above). To find the square root of a number between 10 and 100 the right half of the A scale is used. For example, if the hairline is set over 16 on the right half of the A scale (near the middle of the rule), the square root of 16, or 4, is found below it on the D scale.

In general, to find the square root of any number with an odd number of digits or zeros (1, 3, 5, 7, . . .), the left half of the A scale is used. If the number has an even number of digits or zeros (2, 4, 6, 8, . . .), the right half of the A scale is used. In these statements it is assumed that the number is not written in standard form.

Fig. 13



The table below shows the number of digits or zeros in the number N and its square root, and also whether right or left half of the A scale should be used.

	ZEROS				or	DIGITS				
	L R	L R	L R	L	R	L R	L R	L R	L R	
N	7 or 6	5 or 4	3 or 2	1	0	1 or 2	3 or 4	5 or 6	7 or 8	etc.
$\sqrt{N}$	3	2	1	0	0	1	2	3	4	etc.

This shows that numbers from 1 up to 100 have one digit in the square root; numbers from 100 up to 10,000 have two digits in the square root, etc. Numbers which are less than 1 and have, for example, either two or three zeros, have only one zero in the square root. Thus $\sqrt{0.004} = 0.0632$, and $\sqrt{0.0004} = 0.02$.

• EXAMPLES:

(a) Find $\sqrt{248}$. This number has 3 (an *odd* number) digits. Set the hairline of 248 of the left A scale. Therefore the result on D has 2 digits, and is 15.75 approximately.

(b) Find $\sqrt{563000}$. The number has 6 (an *even* number) digits. Set the hairline on 563 of the right A scale. Read the figures of the square root on the D scale as 75. The square root has 3 digits and is 750 approximately.

(c) Find $\sqrt{.00001362}$. The number of *zeros* is 4 (an *even* number). Set the hairline on 1362 of the right half of the A scale. Read the figures 369 on the D Scale. The result has 2 zeros, and is .00369.

If the number is written in standard form, the following rule may be used. If the exponent of 10 is an even number, use the left half of the A scale and multiply the reading on the D scale by 10 to an exponent which is ½ the original. If the exponent of 10 is an odd number, move the decimal point one place to the right and decrease the exponent of 10 by one, then use the right half of the A scale and multiply the reading on the D scale by 10 to an exponent which is ½ the reduced exponent. This rule applies to either positive or negative exponents of 10.

• Examples:

(a) Find the square root of 3.56×10^4. Place hairline of indicator on 3.56 on the left half of the A scale and read 1.887 on the D scale. Then the square root is $1.887 \times 10^2 = 188.7$.

(b) Find the square root of 7.43×10^{-5}. Express the number as 74.3×10^{-6}. Now place the hairline of the indicator over 74.3 on the right half of the A scale and read 8.62 on the D scale. Then the desired square root is 8.62×10^{-3}.

All the above rules and discussion can be applied to the B and C scales if it is more convenient to have the square root on the slide rather than on the body of the rule.

***Squares:* To find the square of a number, reverse the process for finding the square root. Set the indicator over the number on the D scale and read the square of that number on the A scale; or set the indicator over the number on the C scale and read the square on the B scale;**

$$\frac{A \mid n^2 \mid}{D \mid n \mid}, \quad \text{or} \quad \frac{B \mid n^2 \mid}{C \mid n \mid}.$$

• Examples:

(a) Find $(1.73)^2$ or 1.73×1.73. Locate 1.73 on the D scale. On the A scale find the approximate square 3.

(b) Find $(62800)^2$. Locate 628 on the D scale. Find 394 above it on the A scale. The number has 5 digits. Hence the square has either 9 or 10 digits. Since, however, 394 was located on the right half of the A scale, the square has the *even* number of digits, or 10. The result is 3,940,000,000.

(c) Find $(.000254)^2$. On the A scale read 645 above the 254 of the D scale. The number has 3 zeros. Since 645 was located on the side of the A scale for "odd zero" numbers, the result has 7 zeros, and is 0.0000000645.

PROBLEM SET XII

1. $\sqrt{7.3}$	**2.** $\sqrt{73}$	**3.** $\sqrt{841}$
4. $\sqrt{45}$	**5.** $\sqrt{450}$	**6.** $\sqrt{10}$
7. $\sqrt{1.64}$	**8.** $\sqrt{16.4}$	**9.** $\sqrt{0.084}$
10. $(1.91)^2$	**11.** $(19.1)^2$	**12.** $(3.95)^2$
13. $(5.74)^2$	**14.** $(57.4)^2$	**15.** π^2
16. $\sqrt{710}$	**17.** $\sqrt{0.0710}$	**18.** $\sqrt{\pi}$
19. $\sqrt{2.5}$	**20.** $\sqrt{3000}$	**21.** $\sqrt{0.006}$
22. $(48.2)^2$	**23.** $(0.087)^2$	**24.** $(0.00284)^2$
25. $\sqrt{92}$	**26.** $\sqrt{0.062}$	**27.** $\sqrt{0.000,000,94}$
28. $\sqrt{3.73}$	**29.** $\sqrt{26.5}$	**30.** $\sqrt{77}$
31. $(1.66)^2$	**32.** $(8.85)^2$	**33.** $(0.32)^2$
34. $(2.63)^2$	**35.** $(47.9)^2$	**36.** $(0.072)^2$

THE K SCALE: Cube Roots and Cubes

The product of three equal factors is called the *cube* of the factor. For example, $4 \times 4 \times 4 = 64$, and 64 is the cube of 4. The factor 4 is called a *cube root* of 64. It is customary to write $4^3 = 64$ and $\sqrt[3]{64} = 4$ or $64^{\frac{1}{3}} = 4$.

On the slide rule a scale marked with the letter K may be used in finding the cube or cube root of any number.

***Rule:* The cube root of any number located on the K scale is found directly opposite on the D scale;**

$$\frac{K \mid n \mid}{D \mid \sqrt[3]{n} \mid}.$$

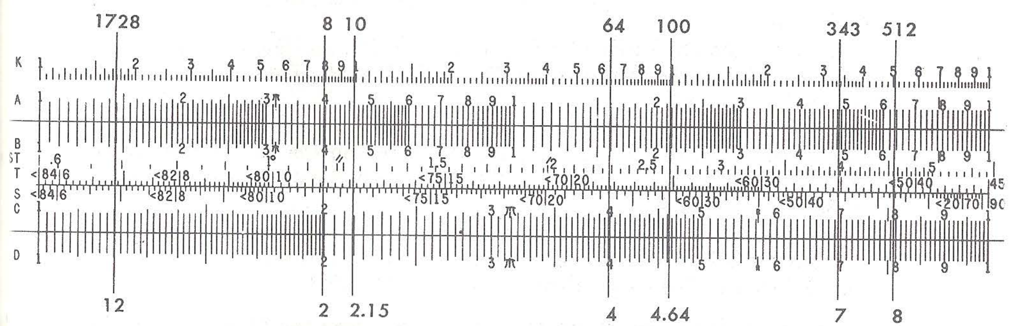
• Example:

Find $\sqrt[3]{8}$. Place the hairline of the indicator over the 8 at the end of the K scale. The cube root, 2, is read directly opposite on the D scale.

Reading the scales: The K scale is a contraction of the D scale itself. The D scale has been shrunk to one third its former length and printed three times on the same line. To find the cube root of any number between 0 and 10 the left third of the K scale is used. To find the cube root of a number between 10 and 100 the middle third is used. To find the cube root of a number between 100 and 1000 the right-hand third of the K scale is used to locate the number.

In general, to decide which part of the K scale to use in locating a number, mark off the digits in groups of three starting from the decimal

Fig. 14



point. If the left group contains one digit, the left third of the K scale is used; if there are two digits in the left group, the middle third of the K scale is used; if there are three digits, the right-hand third of the K scale is used. In other words, numbers containing 1, 4, 7, . . . digits are located on the left third; numbers containing 2, 5, 8, . . . digits are located on the middle third; and numbers containing 3, 6, 9, . . . digits are located on the right third of the K scale. The corresponding number of digits or zeros in the cube roots are shown in the table below and also whether the left, center or right section of the K scale should be used.

	ZEROS				or	DIGITS			
	L C R	L C R	L C R	L C	R	L C R	L C R	L C R	L C R
N	11, 10, 9	8, 7, 6	5, 4, 3	2, 1	0	1, 2, 3	4, 5, 6	7, 8, 9	10, 11, 12
$\sqrt[3]{N}$	3	2	1	0	0	1	2	3	4

• EXAMPLES:

(a) Find $\sqrt[3]{6.4}$. Set hairline over 64 on the left-most third of the K scale. Read 1.857 on the D scale.

(b) Find $\sqrt[3]{64}$. Set hairline over 64 on the middle third of the K scale. Read 4 on the D scale.

(c) Find $\sqrt[3]{640}$. Set hairline over 64 on the right-most third of the K scale. Read 8.62 on the D scale.

(d) Find $\sqrt[3]{6,400}$. Set hairline over 64 on the left third of the K scale. Read 18.57 on the D scale.

(e) Find $\sqrt[3]{64,000}$. Set hairline over 64 on the middle third of the K scale. Read 40 on the D scale.

(f) Find $\sqrt[3]{0.0064}$. Use the left third of the K scale, since the first group of three, or 0.006, has only one non-zero digit. The D scale reading is then 0.1857.

(g) Find $\sqrt[3]{0.064}$. Use the middle third of the K scale, reading 0.4 on D.

If the number is expressed in standard form it can either be written in ordinary form or the cube root can be found by the following rule.

***Rule:* Make the exponent of 10 a multiple of three, and locate the number on the proper third of the K scale. Read the result on the D scale and multiply this result by 10 to an exponent which is ⅓ the former exponent of 10.**

• EXAMPLES:

(a) Find the cube root of 6.9×10^3. Place the hairline over 6.9 on the left third of the K scale and read 1.904 on the D scale. Thus the desired cube root is 1.904×10^1.

(b) Find the cube root of 4.85×10^7. Express the number as 48.5×10^6 and place the hairline of the indicator over 48.5 on the middle third of the K scale. Read 3.65 on the D scale. Thus the desired cube root is 3.65×10^2. or 365.

(c) Find the cube root of 1.33×10^{-4}. Express the number as 133×10^{-6} and place the hairline over 133 on the right third of the K scale. Read 5.10 on the D scale. The required cube root is 5.10×10^{-2}.

***Cubes:* To find the cube of a number, reverse the process for finding cube root. Locate the number on the D scale and read the cube of that number on the K scale;**

K	n^3
D	n

• EXAMPLES:

(a) Find $(1.37)^3$. Set the indicator on 1.37 of the D scale. Read 2.57 on the K scale.

(b) Find $(13.7)^3$. The setting is the same as in example (a), but the K scale reading is 2570, or 1000 times the former reading.

(c) Find $(2.9)^3$ and $(29)^3$. When the indicator is on 2.9 of D, the K scale reading is 24.4. The result for 29^3 is therefore 24,400.

(d) Find $(6.3)^3$. When the indicator is on 6.3 of D, the K scale reading is 250.

PROBLEM SET XIII

1. $\sqrt[3]{4}$	**2.** $\sqrt[3]{40}$	**3.** $\sqrt[3]{400}$
4. $\sqrt[3]{0.004}$	**5.** $\sqrt[3]{.04}$	**6.** $\sqrt[3]{0.4}$
7. 16^3	**8.** 1.6^3	**9.** $(0.16)^3$
10. 17^3	**11.** 2.45^3	**12.** $(0.738)^3$
13. 164.5^3	**14.** 56.1^3	**15.** π^3
16. $\sqrt[3]{5.4}$	**17.** $\sqrt[3]{71.}$	**18.** $\sqrt[3]{815}$
19. $\sqrt[3]{0.0315}$	**20.** $\sqrt[3]{525,000}$	**21.** $\sqrt[3]{0.156}$
22. $(0.933)^3$	**23.** $(0.62)^3$	**24.** $(0.021)^3$
25. $(0.47)^3$	**26.** $(0.15)^3$	**27.** $(0.083)^3$
28. $\sqrt[3]{0.000,51}$	**29.** $\sqrt[3]{53}$	**30.** $\sqrt[3]{\pi}$
31. $\sqrt[3]{3,700}$	**32.** $\sqrt[3]{85,000}$	**33.** $\sqrt[3]{750,000}$

COMBINED OPERATIONS: Square Roots and Squares

The A and B scales may be used for multiplication or division in exactly the same way as the C and D scales. Since the scales are shorter, there is some loss in accuracy. Nevertheless, most computers employ the A and B scales (in conjunction with the C and D scales) to avoid extra steps which would also lead to loss of accuracy.

Many problems involve expressions like $\sqrt{ab}$ or ab^2, etc. With a little care, many such problems involving combined operations may be easily computed. The list of possibilities is extensive. Only a few examples will be given.

One of the basic operations with square roots is indicated by $P = a\sqrt{b}$. The setting is as follows:

B		b
C	1	
D	a	P

• EXAMPLES:

(a) Find $2\sqrt{3}$. Set 1 of C over 2 of D. Move hairline over 3 on B, read the result 3.46 on D.

(b) Find $3.27 \times \sqrt{48.6}$. Set right-hand 1 of C over 3.27 on D. Move hairline to 48.6 on right-hand part of the B scale. Read 22.8 on D. Or, set right-hand 1 of B under 48.6 on A. Move indicator to 3.27 on C. Read 22.8 on D. Or, move hairline to 48.6 on A. Move slide so 3.27 on CI is under the hairline. Read 22.8 on D at the index of C.

A second basic operation is indicated by $Q = a \div \sqrt{b}$. The setting is as follows:

B	b	
C		1
D	a	Q

By combining these two basic operations various examples are easily computed.

• Examples:

(a) Find $8 \div \sqrt{5}$. Set indicator over 8 on D. Move slide so 5 on B is under hairline. At index of C read 3.58 on D.

(b) Find $13 \div \sqrt{62.9}$. Set hairline over 13 on D. Move slide so 62.9 on right-hand B scale is under the hairline. At 1 of C read 1.639 on D.

(c) $\dfrac{24.6\sqrt{15.2}}{\sqrt{4.93}}$. Set the hairline over 24.6 on D. Move slide so 4.93 on B is under the hairline. Move the hairline to 15.2 on right-hand B. Read 43.3 on D.

(d) $\dfrac{62 \times \sqrt{1.9} \times \sqrt{30.4}}{\sqrt{54} \times 28.3}$. Set the hairline over 62 on D. Move slide so 54 on right-hand B is under the hairline. Move indicator to 1.9 on B. Move slide so 28.3 on C is under the hairline. Move hairline to 30.4 on B. Read 2.265 on D.

The corresponding operations with squares, namely $a \times b^2$ and $a \div b^2$, can be done as straight-forward multiplication or division. They may also be done by reversing the methods used with square roots. When this is done, the roles of the scales are reversed as follows:

the A scale is used instead of D;
the B scale is used instead of C;
the C scale is used instead of B.

Thus for $P = ab^2$ and $Q = a \div b^2$ the setting for P is

A	a	P
B	1	
C		b

; and for Q it is

A	a	Q
B		1
C	b	

• Examples:

(a) Find $P = 2 \times 3^2$. Set 1 of B under 2 on A. Move hairline to 3 on C. Read P = 18 on A.

(b) Find $Q = 18 \div 3^2$. Set hairline over 18 on A. Move slide so 3 on C is under hairline. At 1 of B read Q = 2 on A.

(c) Find 1.63×5.41^2. Set the left index on B under 1.63 of A. Move the indicator to 5.41 on C. Read the result 47.7 under the hairline on A.

PROBLEM SET XIV

1. $6\sqrt{5}$
2. $4\sqrt{7}$
3. $24.7\sqrt{8.21}$
4. $13.5\sqrt{9.62}$
5. $8 \times (2.3)^2$
6. $6 \times (1.79)^2$
7. $37.4 \times (18.1)^2$
8. $162 \times (0.45)^2$
9. $32 \div \sqrt{120}$
10. $27/\sqrt{141}$
11. $625 \div \sqrt{6.74}$
12. $8/\sqrt{0.0441}$
13. $\dfrac{8.3 \times \sqrt{13}}{\sqrt{7}}$
14. $\dfrac{146\sqrt{5.41}}{\sqrt{23}}$
15. 2.83×8.19^2
16. $73 \times (0.86)^2$
17. $4.21 \div 5.34^2$
18. $425 \div (16.2)^2$
19. $\dfrac{0.42\sqrt{78.1}}{3.92}$
20. $\dfrac{0.95\sqrt{0.092}}{8.61}$
21. $\dfrac{\pi\sqrt{228} \times \sqrt{53}}{2 \times \sqrt{605}}$
22. $\dfrac{19\sqrt{0.76} \times \sqrt{8.1}}{\sqrt{9.4} \times \sqrt{67}}$
23. $\sqrt{3.25 \times 4.18}$
24. $\sqrt{8.36 \times 19.1}$

COMBINED OPERATIONS: Cube Roots and Cubes

Combined operations with cubes and cube roots are done by methods similar to those for squares and square roots. However, since there is no K scale on the slide, there is less flexibility in choice of methods.

• Examples:

(a) Find $4\sqrt[3]{13}$. Set the hairline over 13 on the middle section of the K scale, and set the slide so 1 of C is under the hairline. Move hairline to 4 on C. Read result on D as 9.40.

(b) Find $\sqrt[3]{700} \div 3.25$. Set hairline over 7 on the right-hand section of K. Set 3.25 on C under the hairline. At 1 of C read 2.73 on D.

(c) Find $8 \div \sqrt[3]{2}$. Think of this as $1/(\sqrt[3]{2}/8)$. Set the hairline over 2 on the left-hand section of K. Move slide so 8 on C is under the hairline. Move the hairline over 1 on C. Read 6.35 on DI. If your slide rule has no DI scale, you can read the answer on C above 1 on D.

PROBLEM SET XV

1. $6\sqrt[3]{9}$
2. $5\sqrt[3]{4}$
3. $29.3\sqrt[3]{3.1}$
4. $18.6\sqrt[3]{15}$
5. $\sqrt[3]{56} \div 2.19$
6. $\sqrt[3]{8.94} \div 0.278$
7. $\sqrt[3]{209} \div 5.48$
8. $\sqrt[3]{65} \div 4.09$
9. $(3.4 \times 6.7)^3$
10. $\left(\dfrac{22.6 \times 4.79}{12.5}\right)^3$

11. $(4.1)^3 \times (2.8)^3$

12. $16.3^3 \div (3.2)^3$

13. $19 \div \sqrt[3]{28}$

14. $306 \div \sqrt[3]{7}$

15. $41.5 \div \sqrt[3]{698}$

16. $\pi \div \sqrt[3]{10}$

17. $\dfrac{98.6 \times \sqrt[3]{78}}{32.2}$

18. $\dfrac{0.083 \times \sqrt[3]{.0082}}{0.0092}$

19. $(3.5 \times 6.7)/\sqrt[3]{6.5}$

20. $\sqrt[3]{34}/(7.63 \times 0.24)$

21. $(\sqrt[3]{1.8} \times \sqrt{215})/92$

22. $(\sqrt[3]{39.2} \times \sqrt{0.26})/0.43$

23. $\dfrac{\sqrt[3]{497} \times \sqrt{1.56} \times \pi}{\sqrt{12.3}}$

PRACTICAL PROBLEMS, SET XVI

1. Find the area of a circular skylight whose radius is 1 ft., 8 in. The formula is $A = \pi r^2$ or $A = 3.14 \times (1.67)^2$.
2. Find the volume of a concrete slab whose dimensions are $8'' \times 24' \times 36'$.
3. Find the volume of a cube whose edge is 3 ft., 9 in., or 3.75 ft.
4. Find the length of one side of a square which contains one acre, or 43,560 sq. ft., that is, find $\sqrt{43{,}560}$.
5. Use the formula $S = \frac{1}{2}gt^2$ to calculate how far a freely falling object will fall in 8.5 seconds. Use 32.2 ft. per second per second for g.
6. Estimate the interest on \$860 at $2\frac{1}{2}\%$ for 45 days. Here $$I = \frac{860 \times .025}{8}.$$
7. Find the edge of a cubical box which is to contain 180 cubic inches. $\sqrt[3]{180} = ?$
8. Find the diagonal of a cube whose edge is 24 inches. Here $d = 24\sqrt{3}$.
9. In a basketball game 93 points were scored by one team, as follows: Joe, 17; Bill, 21; Tom, 14; Henry 8; Dick, 12; Al, 9; Bob, 7; and Mark, 5. With only one setting of the slide, find what per cent of the total was scored by each player.
10. At 2.25%, what would be the interest on \$680 for one year?
11. If the voltage is 110 and the current is 2.3 amps, what is the resistance?

THE L SCALE: Logarithms

The L scale is used to find logarithms to the base 10, or common logarithms. The logarithm of a number is the exponent to which a given base must be raised to produce the number. For example, $\log 10^2 = 2.000$; $\log 10^3 = 3.000$, etc. A logarithm consists of two parts. The integral part (on the left of the decimal point) is called the *characteristic*.

The *mantissa* is the decimal fraction part on the right of the decimal point. The L scale is used for finding the mantissa of the logarithm (to the base 10) of any number. The mantissa of the logarithm is the same for any series of digits regardless of the location of the decimal point.

The position of the decimal point in the given number determines the characteristic of the logarithm, and conversely. The rule given on page 10 may be used to determine the characteristic. However, some people prefer the following rules to determine the characteristic.

1. For 1, and all numbers greater than 1, the characteristic is one less than the number of places to the left of the decimal point in the given number.
2. For numbers smaller than 1 (that is, for decimal fractions) the characteristic is negative. Its numerical value is one more than the number of zeros between the decimal point and the first significant figure in the given number.

***Rule:* Locate the number on the D scale (when L scale is on the body), and read the mantissa of its logarithm (to the base 10) on the L scale. Determine the characteristic. If the L scale is on the slide, use the C scale instead of the D scale.**

• Example:
Find the logarithm of 425. Set the hairline over 425 on the D scale. Read the mantissa of the logarithm (.628) on the L scale. Since the number 425 has 3 digits, the characteristic is 2 and the logarithm is 2.628.

If the logarithm of a number is known, the number may be found by reversing the above process. The characteristic is ignored until the decimal point is to be placed in the number.

• Examples:
(a) Find x, if $\log x = 3.248$. Set the hairline over 248 on the L scale. Above it read the number 177 on the D scale. Since the characteristic is 3, there are 4 digits in the number, $x = 1770$.
(b) Find the logarithm of 0.000627. Opposite 627 on the D scale find .797 on the L scale. Since the number has 3 zeros, the characteristic is -4 and the logarithm is $-4 + .797$ or -3.203.

Note that the mantissa of a logarithm is always positive but the characteristic may be either positive or negative. In computations, negative characteristics are troublesome and frequently are a source of error. It is customary to handle the difficulty by not actually combining the negative characteristic and positive mantissa. For example, if the characteristic is -4 and the mantissa is .797, the logarithm may be written $0.797 - 4$. This same number may also be written $6.797 - 10$, or $5.797 - 9$, and in other ways as convenient. In each of these forms if the integral parts are combined, the result is -4. Thus $0 - 4 = -4$; $6. - 10 = -4$; $5. - 9 = -4$. The form which shows that the number 10 is to be subtracted is the most common.

• Examples: $\log 0.4 = 9.602 - 10$, $\log 0.0004 = 6.602 - 10$.

PROBLEM SET XVII

Use the L scale to find the following.

1. log 547	**2.** log 0.00124
3. log 22,800	**4.** log 0.678
5. log 3.26	**6.** log 0.0194
7. log 39.6	**8.** log 0.0014
9. log 10,000	**10.** log 0.000,01

Find x if log x has the following values.

11. 2.699	**12.** 0.916
13. 3.746	**14.** 7.630 − 10
15. 2.825	**16.** 9.357 − 10
17. 8.288 − 10	**18.** 2.866
19. 2.052	**20.** 9.831 − 10

THE S, T, AND ST SCALES: Trigonometry

Introduction

The branch of mathematics called *trigonometry* arose historically in connection with the measurement of triangles. Now, however, it has many other uses in various scientific fields.

There are six basic trigonometric relations. Each relation involves a set of number pairs. A few pairs that belong to one of these relations are shown in the adjoining table. Suppose x represents a given number. Then for each of the six basic relations the function values have special names as follows:

sin x, read "sine x";
cos x, read "cosine x";
tan x, read "tangent x";
cot x, read "cotangent x";
sec x, read "secant x";
cosec x, read "cosecant x".

Number x	Function value *sin x*
0	0
0.5	0.479
1.0	0.841
1.5	0.997

Table 1

Books of tables are often used to find an approximate function value that belongs to a given number. For example, if $x = 0.40$, then cos x or cos 0.40 = 0.921 can be found in a table made for this purpose.

The S, T, and ST scales on a slide rule are used to find approximate function values and to compute with them. The use of these scales results in a great saving of time and work in solving many different kinds of problems.

The commonest use of trigonometric function values is in connection with the measurement of right triangles. In this application the number x is the measure of one of the acute angles. The *unit* of measure may be either the *radian* or the angular *degree.* In practical work the degree is used much more frequently than the radian. Slide rule scales are therefore graduated in *degrees* and decimal fractions of degrees.

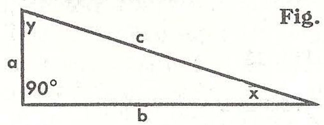


Fig. 14

In Table 1 the numeral 0.5 expresses radians. The corresponding number of degrees is $0.5 \times 180/\pi = 28.6$. The function value is the same as before, and it is customary to write sin 28.6° = 0.479. For right triangles described by the notation used in Fig. 14, the value sin x is equal to the quotient obtained by dividing the length a by the length c; that is, sin $x = a/c$. If any two of the three numbers a, c, and sin x are known, the third can be found by a simple slide rule computation. The list of basic relations for right triangles is as follows.

$\sin x = a/c$,	$\operatorname{cosec} x = c/a$,
$\cos x = b/c$,	$\sec x = c/b$,
$\tan x = a/b$,	$\cot x = b/a$.

In books on trigonometry hundreds of other relations are discussed. A few of these will be used in later sections of this manual.

THE S SCALE: Sines and Cosines

The scale marked S is used in finding the approximate value of the sine or cosine of any angle between 5.7 degrees and 90 degrees. Since $\sin x = \cos (90 - x)$, the same graduations serve for both sines and cosines. Thus

$$\sin 6° = \cos (90° - 6°) = \cos 84°.$$

The numerals printed at the right of the longer graduations are read when sines are to be found. Those printed at the left are used when cosines are to be found. On the slide rule, angles are divided decimally instead of into minutes and seconds. Thus sin 12.7° is represented by the 7th small graduation to the right of the mark for 78|12.

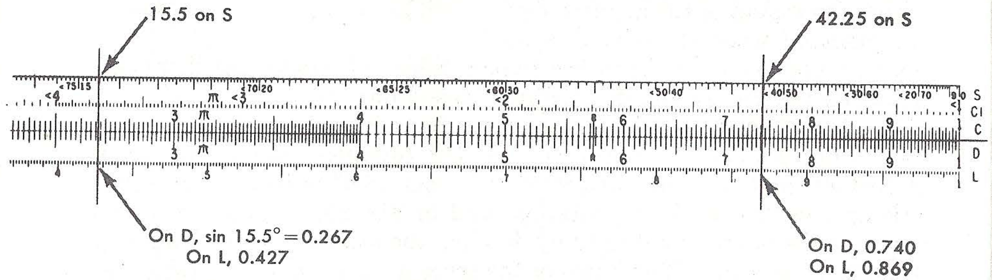
Sines (or cosines) of all angles on the S scale have the decimal point at the left of figures read from the C (or D) scale.

Rule: **To find the sine of an angle on the S scale, set the hairline on the graduation which represents the angle. (Remember to read sines from left to right using the numerals to the right of the graduation.) Read the sine on the C scale under the hairline. If the slide is placed so the C and D scales are exactly together, the sine can also be read on the D scale, and the mantissa of the logarithm of the sine (log sin) may be read on the L scale.**

- EXAMPLE:
  Find sin 15° 30′ and log sin 15° 30′. Set left index of C scale over left index

of D scale. Set hairline over 15.5° (i.e., 15° 30′) on the S scale. Read sin 15.5° = 0.267 on C or D scale. Read mantissa of log sin 15.5° = 0.427 on L

Fig. 15



scale. According to the rule for characteristics of logarithms, this would be 9.427 − 10.

Rule: **To find the cosine of an angle on the S scale, set the hairline on the graduation which represents the angle. (Remember to read from right to left using the numerals to the left of the graduations.) Read the cosine on the C scale under the hairline. If the slide is placed so the C and D scales are exactly together, the cosine can also be read on the D scale, and the mantissa of the cosine (log cos) may be read on the L scale.**

- EXAMPLE:
  Find cos 42° 15′ and log cos 42° 15′. Set left index of C scale over left index of D scale. Set hairline on 42.25° (i.e., 42° 15′) on the S scale. Read cos 42.25° = 0.740 on C or D scale. Read mantissa of log cos 42.25° = 0.869 on L scale. According to rule for characteristics of logarithms, this would be 9.869 − 10.

Finding the Angle.

If the trigonometric function value is known, and the size of the angle less than 90° is to be found, the above rules are reversed. The function value is set on the C scale, and the angle itself read on the S scale.

- EXAMPLES:
  (a) Given sin x = 0.465, find x. Set indicator on 465 of C scale, read x = 27.7° on the S scale.
  (b) Given cos x = 0.289, find x. Set indicator on 289 on C scale. Read x = 73.2° on the S scale.

If sec x is to be found, the relation sec x = 1/cos x is used. The number x is set on the S scale, reading from right to left, and sec x is read on the CI scale.

If csc x is to be found, the relation csc x = 1/sin x is used. The number x is set on the S scale, and csc x is read on the CI scale.

PROBLEM SET XVIII

A table in the form below can be completed if any one of the seven numbers x, sin x, cos x, etc. is known. The "Problem number" and one of the seven numbers are given. Draw up such a table and complete it. The columns headed tan x and cot x may be left blank until the section of the manual on the T scale has been studied. The 12 problems in this set follow the table.

Problem Number	x	sin x	cos x	tan x	cot x	sec x	csc x
1.	16.5°	0.284	0.959	0.296	3.38	1.043	3.52
2.	20°						
3.		0.866					
4.	42°						
5.			0.50				

1. x = 16.5°
2. x = 20°
3. sin x = 0.866
4. x = 42°
5. cos x = 0.286
6. x = 68°
7. x = 11.72°
8. sin x = 0.312
9. cos x = 0.893
10. sin x = 0.635
11. csc x = 1.124
12. sec x = 6.76

THE T SCALE: Tangents and Cotangents

The T scale, together with the C or CI scales, is used to find the value of the tangent or cotangent of angles between 5.7° and 84.3°. Since tan x = cot (90 − x), the same graduations serve for both tangents and cotangents. For example, if the indicator is set on the graduation marked 60|30, the corresponding reading on the C scale is .577, the value of tan 30°. This is also the value of cot 60°, since tan 30° = cot (90° − 30°) = cot 60°. Moreover, tan x = 1/cot x; in other words, the tangent and cotangent of the same angle are reciprocals. Thus for the same setting, the reciprocal of cot 60°, or 1/.577, may be read on the CI scale as 1.732. This is the value of tan 60. These relations lead to the following rule.

Rule: **Set the angle value on the T scale and read**
(i) tangents of angles from 5.7° to 45° on C,
(ii) tangents of angles from 45° to 84.3° on CI,
(iii) cotangents of angles from 45° to 84.3° on C,
(iv) cotangents of angles from 5.7° to 45° on CI.

If the slide is set so that the C and D scales coincide, these values may also be read on the D scale. Care must be taken to note that the T scale readings for angles between 45° and 84.3° *increase from right to left.*

In case (i) above, the tangent values are all between 0.1 and 1.0; that is, the decimal point is at the left of the number as read from the C scale.

In case (ii), the tangents are greater than 1.0, and the decimal point is placed to the right of the first digit as read from the CI scale. For the cotangent values in cases (iii) and (iv) the situation is *reversed.* Cotangents for angles between 45° and 84.3° have the decimal point at the left of the number read from the C scale. For angles between 5.7° and 45° the cotangent is greater than 1 and the decimal point is to the right of the first digit read on the CI scale. These facts may be summarized as follows.

***Rule:* If the tangent or cotangent value is read from the C scale, the decimal point is at the left of the first digit read. If the value is read from the CI scale, it is at the right of the first digit read.**

• EXAMPLES:

(a) Find tan x and cot x when $x = 9°50'$. First note that $50' = \frac{50}{60}$ of 1 degree = 0.83°, approximately. Hence 9° 50′ = 9.83°. Locate $x = 9.83°$ on the T scale. Read tan $x = 0.173$ on the C scale, and read cot $x = 5.77$ on the CI scale.

(b) Find tan x and cot x when $x = 68.6°$. Locate $x = 68.6°$ on the T scale reading from right to left. Read 255 on the CI scale. Since all angles greater than 45° have tangents greater than 1 (that is, have one digit as defined above), tan $x = 2.55$. Read cot 68.6° = 0.392 on the C scale.

Finding the Angle.

If the trigonometric function value is known, and the size of the angle less than 90° is to be found, the above rules are reversed. The value is set on the C or CI scale, and the angle itself read on the T scale.

• EXAMPLES:

(a) Given tan $x = 0.324$, find x. Set 324 on the C scale, read 17.9° on the T scale.

(b) Given tan $x = 2.66$, find x. Set 266 on the CI scale, read $x = 69.4°$ on the T scale.

(c) Given cot $x = 0.630$, find x. Set 0.630 on the C scale, read $x = 57.8°$ on the T scale.

(d) Given cot $x = 1.865$, find x. Set 1865 on the CI scale, read 28.2° on the T scale.

PROBLEM SET XIX

Given the following function values, find the value of x in each example.

1. tan $x = 0.337$ **2.** tan $x = 2.29$
3. cot $x = 1.619$ **4.** cot $x = 0.1078$
5. tan $x = 1.173$ **6.** cot $x = 0.387$
7. sin $x = 0.167$ **8.** sin $x = 0.605$
9. sin $x = 0.1737$ **10.** sin $x = 0.980$
11. sin $x = 0.472$ **12.** cos $x = 0.982$
13. cos $x = 0.317$ **14.** cos $x = .242$

15. cos $x = 0.977$ **16.** cos $x = 0.878$
17. cos $x = 0.794$ **18.** tan $x = 0.164$
19. tan $x = 1.342$ **20.** cot $x = 2.37$
21. cot $x = 0.910$ **22.** tan $x = 1.062$
23. tan $x = 0.1062$ **24.** cot $x = 9.01$
25. cot $x = 5.5$ **26.** tan $x = 5.5$

THE ST SCALE: Small Angles

The sine and the tangent of angles of less than about 5.7° are so nearly equal that a single scale, marked ST, may be used for both. The graduation for 1° is marked with the degree symbol (°). To the left of it the primary graduations represent tenths of a degree. The graduation for 2° is just about in the center of the slide. The graduations for 1.5° and 2.5° are also numbered.

***Rule:* For small angles, set the indicator over the graduation for the angle on the ST scale, then read the value of the sine or tangent on the C scale. Sines or tangents of angles on the ST scale have one zero.**

• EXAMPLES:

(a) Find sin 2° and tan 2°. Set the indicator on the graduation for 2° on the ST scale. Read sin 2° = .0349 on the C scale. This is also the value of tan 2° correct to three digits.

(b) Find sin 0.94° and tan 0.94°. Set the indicator on 0.94 of ST. Read sin 0.94° = tan 0.94° = .0164 on the C scale.

The value of cos x for x small may be found by computing $1 - 2\sin^2(x/2)$. Since cot $x = 1/\tan x$, the cotangents of small angles may be read on the CI scale. Moreover, tangents of angles between 84.3° and 89.42° can be found by use of the relation $\tan x = \cot(90 - x)$. Thus cot 2° = 1/tan 2° = 28.6, and tan 88° = cot 2° = 28.6. Finally, it may be noted that csc $x = 1/\sin x$, and sec $x = 1/\cos x$. Hence the value of these for small angles may be readily found if they are needed. Functions of angles greater than 90° may be converted to equivalent (except for sign) functions in the first quadrant.

• EXAMPLES:

(a) Find cot 1.41° and tan 88.59°. Set indicator at 1.41° on ST. Read cot 1.41° = tan 88.59° = 40.7 on CI.

(b) Find csc 2.18° and sec 2.18°. Set indicator on 2.18° on ST scale. Read csc 2.18° = 1/sin 2.18° = 26.2 on CI. The value of cos 2.18 is approximately 1. Hence sec 2.18 = 1 also.

When the angle is less than 0.57° the approximate value of the sine or tangent can be obtained directly from the C scale by the following procedure. Read the ST scale as though the decimal point were at the left of the numbers printed, and read the C scale (or D, CI, etc.) with the decimal point one place to the left of where it would normally be. Thus sin 0.2° = 0.00349; tan 0.16° = 0.00279, read on the C scale.

PROBLEM SET XX

Use the slide rule scales to find the following.

1. sin 1.4°	**2.** tan 2.8°
3. sin 0.82°	**4.** cot 0.78°
5. sin 3.64°	**6.** sin 0.040°
7. cot 3.92°	**8.** tan 0.035°
9. cos 1.2°	**10.** csc 1.5°
11. x if $\sin x = 0.0227$	**12.** x if $\tan x = 0.0405$
13. y if $\sin y = 0.0541$	**14.** x if $\cot x = 22.9$
15. t if $\sin t = 0.0123$	**16.** y if $\tan y = 0.0163$
17. x if $\cot x = 35.8$	**18.** x if $\csc x = 77$
19. t if $\sec t = 1.004$	**20.** y if $\csc y = 12.7$

SPECIAL GRADUATIONS

For many years certain special graduation marks have been included on slide rules. The most obvious of these is the one for π found on the basic scales C, D, etc. This graduation helps in setting π quickly and precisely.

Another special graduation represents $\pi/4$. It is placed on the A, B, C and D scales at 0.7854, near the right-hand end. It is useful in finding the area of circles when the diameter is known. The area of a circle expressed in terms of the diameter is $A = \frac{\pi}{4} D^2$. Hence the following rule may be used.

Rule: **To find the area of a circle, set the index of B to 0.7854 on A. Move the hairline to the diameter on C. Read the area on A. To find the diameter when the area is given, set the slide in the same way, then set the hairline over the area on A, and read the diameter on C.**

- EXAMPLES:
  (a) Find the area of circles whose diameters are 3, 45.2, and 8.36. Set right index of B under 0.7854 on A. Move hairline to 3 on C. Read 7.07 on A. Move hairline to 45.2 on C, read 1600 on A. Move hairline to 8.36 on C, read 54.9 on A.
  (b) Find the diameter of a circle whose area is 430. Set 1 of B to 0.7854 on A. Move hairline to 430 on the left-hand A scale. Read 23.4 on C.

A special graduation located at 57.3 on the C, D, and CI scales is indicated by the letter R. This graduation is useful in changing from radian measure to degree measure, and conversely. Since 1 radian is equal to 57.3 degrees, approximately, the following rule may be used.

Rule: **When 1 of C is set over any number of radians on D, under R of C read the corresponding number of degrees on D, and conversely. Also, when R on C is set over 1 on D, the number of degrees on C is opposite the number of radians on D, and conversely.**

- EXAMPLES:
  (a) Find the number of degrees in 2.48 radians. Set 1 of C over 248 on D. Under R on C read 142° on D.
  (b) Find the number of radians in an angle of 36.7°. Set R on C over 36.7 on D. Under 1 of C read 0.642 radians on D.
  (c) Find the number of degrees in $\pi/4$ radians. Set 4 on C over π on D. Under R on C read 45° on D.

Two seldom used special graduations are placed on the ST scale. One is indicated by a longer graduation found just to the left of the graduation for 2° at about 1.97°. When this graduation is set opposite any number of minutes on the D scale, the corresponding number of radians may be read on the D scale under the C index. Also, since $\sin x = x$ and $\tan x = x$, for x in radians and small, the value of the sine (or the tangent) of an angle of that many minutes is found by the same setting.

Sin 0° = 0, and sin 1′ = 0.00029, and for small angles the sine increases by 0.0029 for each increase of 1′ in the angle. Thus sin 2′ = 0.00058; sin 3.44′ = 0.00100, and the sines of all angles between 3.44′ and 34.4′ have two zeros. Sines of angles between 34.4′ and 344′ (or 5.73°) have one zero. The tangents of these small angles are very nearly equal to the sines.

- EXAMPLE:
  Find sin 6′. With the hairline set the "minute graduation" opposite 6 located on the D scale. Read 175 on the D scale under the C index. Then sin 6′ = 0.00175.

Another special graduation is used to find the sine or tangent of angles expressed in seconds. This graduation is located at about 1.18°. It is used in exactly the same way as the graduation for minutes. Sin 1″ = 0.0000048, approximately, and the sine increases by this amount for each increase of 1″ in the angle, reaching 0.00029 for sin 60″ or sin 1′ = .00029.

PROBLEM SET XXI

1. Find the area of circles whose diameters are as follows: 2.88, 39.2, 150.6, 0.63, 9.25.
2. Find the diameter of circles whose areas are as follows: 18.6, 240, 0.93, 3,460, 82.9.
3. Change the following from radian measure to degrees: 0.3, 1.2, 2.62, 7.8, 0.46, 0.82, 1.73, 5.91, 3.0, 0.04, $\pi/2.4$, $\pi/5.2$, $\pi/8$, 1.5π, 0.8π.
4. For each of the following number of degrees find the corresponding number of radians: 8, 40, 76, 152, 261, 327, 2.3, 5.2, 12.9, 59.5, 110, 190, 202, 308, 350.
5. Change the following number of minutes of angle to radians: 2.3′,

6.5′, 4.82′, 28.1′, 39′. Determine the sine and tangent of each of these angles.

6. Determine the radian measure, sine, and tangent of each of the following number of seconds of angle: 3.6″, 18″, 25″, 38.2″, 56.1″.

COMBINED OPERATIONS

The S, T, and ST scales are on the slide and can therefore be used in combined operations of various types. When used in this way, it is not necessary to *read* the trigonometric function value from the C scale. Instead, only the final answer is read from, for example, the D scale.

• EXAMPLES:

(a) Find $\frac{34.9 \times \sin 27.3^\circ}{8.37}$. Set 837 on C over 349 on D. Move hairline to 27.3 on S. Read 1.912 on D.

(b) Find $\frac{6.2 \times \tan 79}{2.48 \times \cos 38.2}$. Set 2.48 on C over 6.2 on D. Now tan 79 is on CI, so to multiply, move hairline to 1 on C and then move the slide so 79 on T is under the hairline. This must be divided by cos 38.2. Move hairline to 1 on C, and move slide so 38.2 on S (reading from right to left) is under hairline. At index of C, read 16.36 on D.

(c) Find $\frac{8.26 \times \cos 62.3 \times \sqrt{57}}{\tan 32 \times \sin 41.3}$. Set the hairline over 8.26 on D, and move slide so 32 on T is under the hairline. Move hairline to 62.3 on S (reading from right to left). Move slide so 41.3 on S (reading left to right) is under hairline. Move hairline to 57 on B. Read answer as 70.3 on D.

The decimal point is found by the following rough calculation:

$$\frac{8 \times .5 \times 7}{.6 \times .6} \text{ or } \frac{80 \times 5 \times 7}{6 \times 6},$$

which is obviously near 80.

Computations as complicated as this one are rarely needed in practical work. They are included here to show the power of the slide rule as a computational tool, and for the sake of completeness.

PROBLEM SET XXII

1. $18.6 \sin 16.8^\circ$
2. $89/\sin 46.5^\circ$
3. $9.05 \tan 12^\circ$
4. $50.2/\tan 33^\circ$
5. $25.6 \tan 62^\circ$
6. $8.42 \cot 40^\circ$
7. $42.5 \cos 42^\circ$
8. $13.9/\cos 58.2$
9. $4.98 \sin 3.2^\circ$
10. $532 \tan 6'$
11. $69.3/\cot 58^\circ$
12. $13.9 \csc 20^\circ$
13. $\frac{83.4 \times \sin 39^\circ}{36.5}$
14. $\frac{\sqrt{13} \tan 2.8}{18.1}$

15. $\frac{\sin 19^\circ \times \tan 34^\circ}{27}$
16. $3.04 \times \cos 68^\circ \times \sqrt{193}$
17. $\frac{5.68 \cot 18^\circ}{7.2}$
18. $\frac{\pi \tan 75^\circ}{\sin 48^\circ}$
19. $1308 \times \tan 20^\circ \times \cot 40^\circ$
20. $\sqrt{769} \times \sin 40^\circ \times \tan 2.5^\circ$

PART 3. APPLICATIONS OF TRIGONOMETRY

The use of trigonometric function values in finding the length of the sides and size of the angles of triangles is facilitated by the slide rule. Since this manual is not intended to serve as a substitute for a textbook on trigonometry, the discussion which follows will immediately take up various cases that arise in solving general triangles. Right triangles may then be regarded as special cases.

USING THE LAW OF SINES

When the length of one side of any triangle is known, and the measure of the angle opposite that side is also known, the relation commonly called the "law of sines" is used. This relation may be written in the form

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}.$$

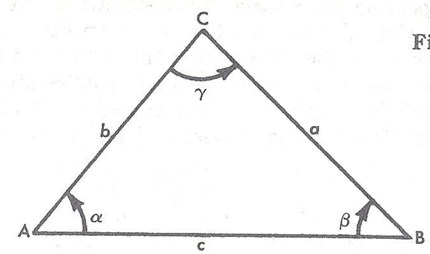


Fig. 16

If one other part is known, the remaining parts may be found by using this proportion. The relation $\alpha + \beta + \gamma = 180$ is also used.

• EXAMPLES:

(a) In triangle ABC, one angle is 35°, the length of the opposite side is 42 feet, and one of the other angles is 70°. Find the size of the remaining angle and the length of the other two sides. First find $\gamma = 75^\circ$ by solving the equation $35 + 70 + \gamma = 180$.

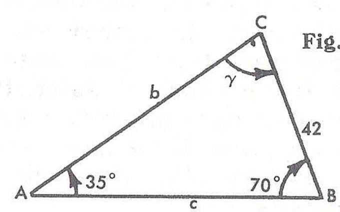


Fig. 17

Then since

$$\frac{\sin 35°}{42} = \frac{\sin 70°}{b} = \frac{\sin 75°}{c}$$

set 35° on S opposite 42 on D. Move the hairline over 70 on S, and read b = 68.8 feet on D. Move the hairline over 75 on S, and read c = 70.7 feet on D. It is not necessary to write out the proportion as was done above,

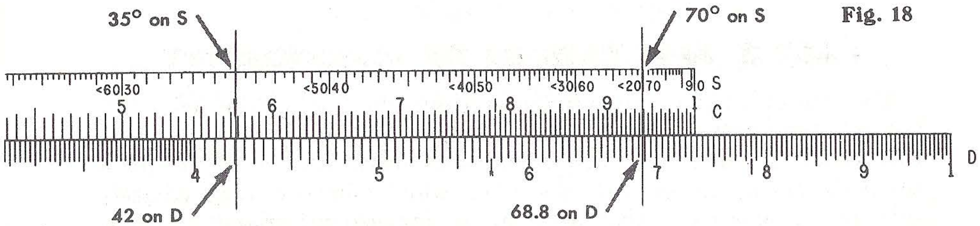


Fig. 18

but it usually is worth the trouble because it helps keep the thinking straight.
(b) Solve the triangle in which b = 60.5, c = 93.8, and γ = 44.7°.

In this example

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{60.5} = \frac{\sin 44.7}{93.8}.$$

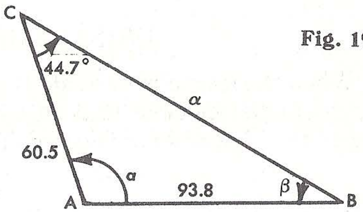


Fig. 19

Set 44.7 on S over 93.8 on D. Move hairline over 60.5 on D, and read β = 27 on S. Then α + 27 + 44.7 = 180, so α = 108.3°. Since 108.3 does not appear on the S scale, it is necessary to set sin (180 − α) = sin 71.7. Move the slide end-for-end; that is, move the hairline to the left index of C and then move the slide so that the right hand index of C is under the hairline. Then move the hairline over 71.7 on S, and read a = 126.6 on D.

(c) One angle of a right triangle is 68.3°, and the adjacent side is 18.6 feet. Find the hypotenuse and the other side.

Here $\quad \dfrac{\sin 68.3}{a} = \dfrac{\sin 21.7}{18.6} = \dfrac{\sin 90}{c}.$

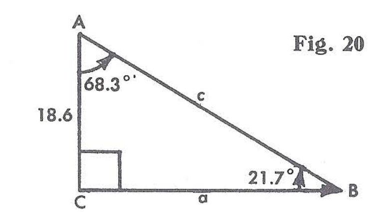


Fig. 20

Recall that sin 90° = 1, so there is no difficulty in setting 90° on the S scale. Set 21.7° on S opposite 18.6 on D. Read c = 50.3 on D under 1 of C. Move indicator to 68.3° on S, read a = 46.7 under the hairline on D.

Certain sets of given data lead to what is commonly called "the ambiguous case" in which two triangles may be found which satisfy the requirements. An analysis of these situations is given in standard trigonometry texts. These cases require no special instruction as to slide rule methods. However, the computer must be on the alert in solving triangles in which a side, the angle opposite it, and another side are given. If the given angle is acute and the side opposite it is less than the other given side, two triangles will usually satisfy the conditions.

PROBLEM SET XXIII

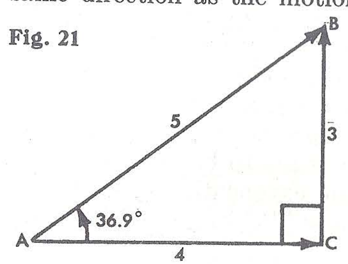
Find the unknown measurements in triangles having known data as follows.

1. $\alpha = 28°$, $\beta = 65°$, $a = 36$	**2.** $a = 25.4°$, $\gamma = 90°$, $c = 48.3$	**3.** $\beta = 20°$, $\gamma = 130°$, $c = 86$
4. $\alpha = 54°$, $\beta = 72°$, $a = 8.96$	**5.** $\beta = 12.6°$, $\gamma = 90°$, $b = 18.4$	**6.** $\gamma = 90°$, $a = 5.21$, $c = 7.63$
7. $\alpha = 18.2°$, $a = 69.3$, $c = 54.2$	**8.** $\beta = 80°$, $b = 37.4$, $c = 37.4$	**9.** $\alpha = 10.5°$, $a = 16$, $b = 24.9$
10. $\alpha = 3.5°$, $\beta = 74.5°$, $c = 846$	**11.** $a = 5°$, $\gamma = 90°$, $a = 0.62$	**12.** $\alpha = 19°$, $a = 27.4$, $b = 84.1$
13. $\alpha = 42.6°$, $\gamma = 90°$, $c = 1432$	**14.** $\gamma = 90°$, $a = 0.24$, $c = 0.48$	**15.** $\alpha = 20°$, $a = 60$, $c = 156$
16. $\alpha = 2°$, $\gamma = 176°$, $a = 3.54$	**17.** $\gamma = 90°$, $a = 1.62$, $c = 24.8$	**18.** $\beta = 45°$, $\gamma = 90°$, $c = 358$
19. $\gamma = 90°$, $a = 2$, $c = 800$	**20.** $a = 70°$, $a = 0.046$, $b = 0.032$	**21.** $\beta = 50.6$, $a = 85.3$, $b = 76.3$

COMPLEX NUMBERS AND VECTORS

A *vector* quantity is one which has both *magnitude* and *direction*. For example, force and velocity are vector quantities. A quantity which has magnitude only is called a *scalar*. For example, mass is a scalar. Vector quantities are often represented by directed straight line segments. The length of the segment represents the magnitude in terms of a selected scale unit. The segment has an initial point A and a terminal point B, and direction is usually indicated by an arrowhead at B pointing in the same direction as the motion of a point which travels from A to B.

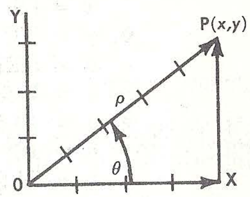
Fig. 21



In Fig. 10, three vectors are represented; namely AB of magnitude 5, AC of magnitude 4, and CB of magnitude 3. Vectors AB and AC have the same initial point, A, and form an angle, CAB, of 36.9°. The initial point of vector CB is at the terminal point of AC. Vectors CB and AB have the same terminal point.

Operations with vectors (for example, addition and multiplication) are performed according to special rules. Thus in Fig. 21, AB may be regarded as the *vector sum* of AC and CB. AB is called the *resultant* of AC and CB: the latter are *components* of AB, and in this case are at right angles to each other. It is frequently desirable to express a given vector in terms of two such components at right angles to each other. Conversely, when the components are given, it may be desirable to replace them with the single resultant vector.

Fig. 22



In algebra, the complex number $x + iy$, where $i = \sqrt{-1}$, is represented by a point P (x, y) in the complex plane, using a coordinate system in which an axis of "pure imaginary" numbers, OY, is at right angles to an axis of "real" numbers, OX.

The same point can be expressed in terms of polar coordinates (ρ, θ) in which the radius vector OP from the origin of coordinates has length ρ and makes an angle θ with the X-axis. The two systems of representation are related to each other by the following formulas:

(1) $x = \rho \cos \theta,$ (3) $\tan \theta = \dfrac{y}{x}$ or $\theta = \arctan \dfrac{y}{x}$

(2) $y = \rho \sin \theta,$ (4) $\rho = \sqrt{x^2 + y^2}$

Finally, the complex number $x + iy$ may be regarded as a vector given in terms of its components x and y and the complex operator $i = \sqrt{-1}$. In practical work the symbol j is preferred to i, to avoid confusion with the symbol often used for the *current* in electricity.

The "Euler identity" $e^{j\theta} = \cos \theta + j \sin \theta$, where θ is expressed in radian measure, can be proved by use of the series expansions of the functions involved. Then $\rho e^{j\theta}$ is an *exponential* representation of the complex number $x + jy$, since $\rho e^{j\theta} = \rho \cos \theta + j\rho \sin \theta = x + jy$. The notation is often simplified by writing $\rho \underline{/\ \theta}$ in place of $\rho e^{j\theta}$.

If two or more complex numbers are to be added or subtracted, it is convenient to have them expressed in the form $x + jy$, since if $N_1 = x_1 + jy_1$, and $N_2 = x_2 + jy_2$, then $N_1 + N_2 = (x_1 + x_2) + j(y_1 + y_2)$. If, however, two or more complex numbers are to be multiplied, it is convenient to have them expressed in the exponential form. Then if $N_1 = \rho_1 e^{j\theta_1}$ and $N_2 = \rho_2 e^{j\theta_2}$, then $N_1 N_2 = \rho_1\rho_2 e^{j(\theta_1 + \theta_2)}$, or $(\rho_1 \underline{/\ \theta_1})\ (\rho_2 \underline{/\ \theta_2}) = \rho_1\rho_2 \underline{/\ \theta_1 + \theta_2}$.

It is therefore necessary to be able to change readily from either of these representations of a complex number to the other.

Right Triangles with Legs Known and Changing from Components to Exponential Form

If a complex number $x + jy$ (or vector in terms of perpendicular components) is given, the problem of changing to the form $\rho \underline{/\ \theta}$ is equivalent to finding the hypotenuse and one acute angle of a right triangle when the lengths of the two legs are known. The formulas $\tan \theta = \dfrac{y}{x}$ and $\rho = y/\sin \theta$, or $\rho = x/\cos \theta$, are the basis of the solution. Thus if $N = 4 + j3$, when 4 of C is set opposite 3 of D, the value of the quotient $\dfrac{y}{x}$, or $\dfrac{3}{4} = .75$ is read on D under the C index. If the indicator is set at the index, and the slide moved so that .75 is under the hairline, the value of $\theta = 36.9°$ may be read on the T scale. Then $\rho = 3/\sin 36.9$ may be computed by moving the indicator to 3 on the D scale, pulling 36.9 on the S scale under the hairline, and reading $\rho = 5$ on the D scale opposite the left index of C. However, this method involves several unnecessary settings and is thus subject to more error than the method given in the general rule below.

Observe that if x and y are both positive and $x = y$, then $\tan \theta = 1$ and $\theta = 45°$. If $y < x$, then $\theta < 45°$; if $y > x$ then $\theta > 45°$. Thus if $y < x$, the T scale is read from *left* to *right*. If $y > x$, the T scale is read from *right* to *left*.

Rule: **(i) To the *larger* of the two numbers (x, y) on D set an index of the slide. Set the indicator over the smaller value on D and read θ on the T scale. If $y < x$, then $\theta < 45$. If $y > x$, then $\theta > 45$, and is read from right to left (or on the left of the graduation mark).**

(ii) Move the slide until θ on scale S is under the indicator, reading S on the same side of the graduation as in (i). Read ρ on D at the index of the C-scale.

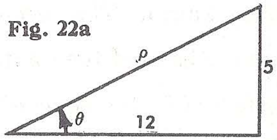
Observe that the reading both begins and ends at an index of the slide. By this method the value of y/x occurs on the C (or CI) scale of the *slide* over the smaller of the two numbers, and the angle may be read immediately on the T scale without moving the slide. In using any method or rule, it is wise to keep a mental picture of the right triangle in mind in order to know whether to read θ on the T or on the ST scale. Thus if y/x is a small number, the angle θ is a small angle, and must be read on the ST scale. To be precise, if $y/x < 0.1$, the ST scale must be used. Similarly, if $y/x > 10$, the angle θ will be larger than 84.3° and cannot be read on the T scale. The complementary angle $\varphi = (90 - \theta)$ will, however, then be on the ST scale, and then θ may be found by subtracting the reading on the ST scale from 90°, since $\theta = 90 - \varphi$.

• **Examples:**

(a) Change $2 + j3.46$ to exponential or "vector" form. Note $\theta > 45$, since $y > x$ (or $3.46 > 2$). Set right index of S opposite 3.46 on D. Move indicator to 2 on D. Read $\theta = 60°$ on T at the *left* of the hairline. Move slide until 60° on scale S is under the hairline (numerals on the *left*), and read $\rho = 4$ on the D scale at the C-index. Then $2 + j3.46 = 4\ \underline{/60°}$.

(b) Find the hypotenuse and both acute angles of a right triangle which has legs of 5 and 12. Set the left index of C over 12 on D. Move the hairline to 5 on D. Read $\theta = 22.6°$ on T. Also read the complementary angle $90 - \theta = 67.4°$ on the T scale by reading from right to left. Move the slide so 22.6° on S is under the hairline. At the index of C read $\rho = 13$ on D.

Fig. 22a



(c) Change $3 + j2$ to exponential or vector form. Note that $\theta < 45°$ since $y < x$ (second component less than first). Set right index of S over 3 on D. Move indicator to 2 on D, read $\theta = 33.7°$ on T (use numerals on the right-hand side of graduations). Move slide so 33.7° on S is under hairline. Read $\rho = 3.60$ on D under index. Hence $3 + j2 = 3.60\ \underline{/33.7}$.

(d) Change $2.34 + j.14$ to exponential form. Since $y < x$, then $\theta < 45°$. Moreover, the ratio y/x is a small number (actually about .06). Since the tangent has one zero, the angle may be read on the ST scale. Set right index of S opposite 2.34 of D. Move indicator to .14 on D. Read $\theta = 3.43°$ on ST. The slide need not be moved. The value of ρ is approximately 2.34. In other words, the angle is so small that the hypotenuse is approximately equal to the longer side. Then $2.34 + j.14 = 2.34\ \underline{/3.43.}$

(e) Change $1.08 + j26.5$ to exponential form. Here $y > x$, so that $\theta > 45°$. But $\dfrac{y}{x} = \dfrac{26.5}{1.08} > 10$. Set right index of S on 26.5 of D. Move indicator to 1.08 of D. Read $\varphi = 2.34°$ on ST. The slide need not be moved. The value of ρ is approximately 26.5; $\theta = 90 - 2.34° = 87.66°$. Hence $26.5\ \underline{/87.66°}$ is the required form.

The following method of changing $x + jy$ to the form $\rho\ \underline{/\theta}$ using the DI scale is sometimes easier to use than methods based on the D scale.

Rule: **(i) To the *smaller* of the two numbers (x, y) on DI set an index of the slide. Set the indicator over the larger value on DI and read θ on the T scale. If $y < x$, then $\theta < 45°$. If $y > x$, then $\theta > 45°$ and is read from right to left (or on the left of the graduation mark).**

(ii) Move the indicator over θ on scale S (or ST), reading S on the same side of the graduation as in (i). Read ρ on DI under the hairline.

• **Examples:**

(a) Change $2 + j\,3.46$ to exponential form. Note that $y > x$ since $3.46 > 2$, and hence $\theta > 45°$. Set right index of C over 2 on DI. Move indicator to 3.46 on DI. Read $\theta = 60°$ on T. Move indicator to 60° on S. Read $\rho = 4$ on DI. Hence $2 + j\,3.46 = 4\underline{/60°}$.

(b) Change $114 + j\,20$ to exponential form. Here $y < x$, so $\theta < 45°$. Set left index of C over 20 on DI. Move indicator to 114 on DI. Read

$\theta = 9.95°$ on T. Move hairline to 9.95° on S. Read $\rho = 116$ on DI. Hence $114 + j\,20 = 116\ \underline{/9.95°}$.

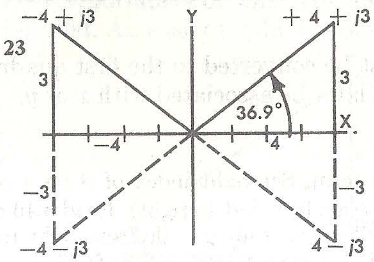
It will be observed that this rule is, in general, easy to use. In step (i) the value of tan θ for $\theta < 45°$ may be observed under the hairline on the C scale, and the value of tan θ for $\theta > 45°$ under the hairline on CI. It may be noted that the rule given first (using the D scale) obtains the result in example (b) above without having the slide project far to the right. Thus, it appears that the relative advantages of the two methods depend in part upon the problem.

If x and y are both positive, $\theta < 90°$. If x and y are *not* both positive, the resultant vector does not lie in the first quadrant, and θ is not an acute angle. In using the slide rule, however, x and y must be treated as both positive. It is therefore necessary to correct θ as is done in trigonometry when an angle is not in the first quadrant.

• **Examples:**

(a) Find the angle between the X-axis and the radius vector for the complex number $-4 + j3$. First solve the problem as though both components were positive. The angle θ obtained is 36.9°. In this case the required angle is $180° - \theta = 180° - 36.9° = 143.1°$. Hence $-4 + j3 = 5\ \underline{/143.1°}$. Similarly for $-4 - j3$, the required angle is $180 + \theta = 180 + 36.9° = 216.9°$, so $-4 - j3 = 5\ \underline{/216.9°}$. For $4 - j3$ the required angle is $360° - \theta = 323.1°$, so $4 - j3 = 5\ \underline{/323.1°}$, which may also be expressed in terms of a negative angle as $5\ \underline{/-36.9°}$.

Fig. 23



(b) Change $17.2 - j6.54$ to exponential form. Here the ratio y/x is negative so θ can be expressed as a negative angle. In numerical value $y < x$, so the numerical or absolute value of $\theta < 45°$. Set left index of S opposite 17.2 on D. Move indicator over 6.54 of D, read $\theta = 20.8°$ on T. Pull 20.8 of S under hairline, read 18.4 on D at left index. Hence $17.2 - j6.54 = 18.4\ \underline{/-20.8°}$, or $18.4\ \underline{/339.2°}$.

PROBLEM SET XXIV

In each of the following problems the given data are the lengths of the legs of a right triangle. Find the missing parts by the methods of this section.

1. $x = 28$, $y = 21$	**2.** $x = 60$, $y = 50$	**3.** $x = 240$, $y = 40$
4. $x = 19.64$, $y = 12.4$	**5.** $x = 30.6$, $y = 78.3$	**6.** $x = 579$, $y = 2410$
7. $x = 40.1$, $y = 37.5$	**8.** $x = 0.207$, $y = 0.421$	**9.** $x = 3650$, $y = 3810$
10. $x = 983.9$, $y = 93.4$		

Change each of the following complex numbers to exponential or polar form.

11. $48 + j77$	**12.** $0.893 + j0.644$
13. $0.0893 + j0.644$	**14.** $16.3 + j0.25$
15. $0.00672 + j0.00385$	**16.** $99.4 + j121.2$
17. $-\ 6.0 + j3.2$	**18.** $-\ 13.4 - j17.2$

Changing from Exponential Form to Components

The process of changing a complex number or vector from the form $\rho e^{j\theta} = \rho \underline{/\theta}$ to the form $x + jy$ depends upon the formulas $x = \rho \cos \theta$, $y = \rho \sin \theta$. These are simple multiplications using the C, D, and S (or ST) scales.

Rule: **Set an index of the S scale opposite ρ on the D scale. Move indicator to θ on the S (or ST) scale, reading from left to right (sines). Read y on the D scale. Move indicator to θ on the S (or ST) scale, reading from right to left (cosines), read x on the D scale.**

If $\theta > 90°$ or $\theta < 0$, it should first be converted to the first quadrant, and the proper negative signs must later be associated with x or y.

• Examples:

(a) Change $4 \underline{/60°}$ to component form. Set right index of S on 4 of D. Move indicator to 60° on S (reading scale from left to right). Read 3.46 on D under hairline. Move indicator to 60° on S, reading scale from right to left (cosines). Read 2 on D under hairline. Hence $4 \underline{/60°} = 2 + j3.46$.

(b) Change $16.3 \underline{/15.4°}$ to the $x + jy$ form. Set left index of S on 16.3 of D. Move indicator to 15.4 of S, read 4.33 on D. Since 15.4° reading from right to left is off the D scale, exchange indices so the right index of C is opposite 16.3 of D. Move indicator to 15.4 of S, and read 15.7 on D. Hence $16.3 \underline{/15.4°} = 15.7 + j4.33$.

(c) Change $7.91 \underline{/3.25°}$ to component form. Set right index of S on 7.91 of D. Move indicator to 3.25 on ST. Read 0.448 on D. To determine the decimal point, observe that the angle is small, and hence the y component will also be small. Obviously, when the hypotenuse is near 8, 4.48 would be too large, and 0.0448 too small, to produce an angle of 3.25°. The cosine cannot be set on ST, but the angle is so small that the x-component is practically equal to the radius vector or hypotenuse. Hence 7.90 is a close approximation; and $7.91 \underline{/3.25°} = 7.90 + j0.448$.

(d) Convert $263 \underline{/160°}$ to the $x = jy$ form. Since $160° > 90°$, compute $180° - 160° = 20°$. Set left index of the S scale on 263 of D. Move indicator to 20° on S. Read 90.0 on D. Move the slide so that the right index of S is on 263 of D. Move indicator to 20 (reading from right to left) on S. Read 247 on D. Since the angle is in the second quadrant, $263 \underline{/160°} = -\ 247 + j90$.

PROBLEM SET XXV

Change the following complex numbers from polar form to component form.

1. $26 \underline{/18°}$	**2.** $21.2 \underline{/15.7°}$
3. $8.35 \underline{/67°}$	**4.** $0.48 \underline{/4.5°}$
5. $396 \underline{/44.5°}$	**6.** $32.9 \underline{/78°}$

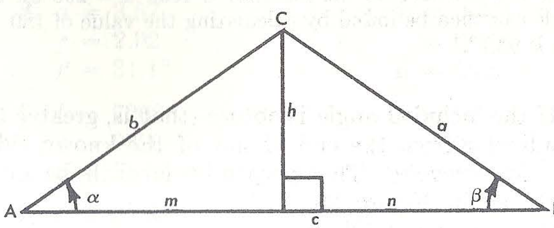
SOLVING TRIANGLES WHEN TWO SIDES AND THE INCLUDED ANGLE ARE KNOWN

When two sides and the included angle of any triangle are known, the relation commonly called the "law of cosines" is sometimes used to find the remaining side. Then the "law of sines" may be used to find the other angles. Thus, in Figure 24, if b, c and α are known

$$a^2 = b^2 + c^2 - 2\,bc \cos \alpha.$$

However, formulas of this type are not convenient when a slide rule is to be used. An easier method proceeds as follows.

Fig. 24



Case (i). Observe that when the included angle measures 90°, the triangle is a right triangle with the two legs known. The methods described above for "Complex Numbers and Vectors" may be used.

Case (ii). If the included angle is acute (that is, less than 90°), draw a perpendicular from the end of the shorter known side to the longer known side. This separates the longer side into two segments and the triangle into two right triangles. Let h denote the length of the perpendicular. Let m denote the length of one segment of the longer side and n the length of the other, and assign m so that the segment it measures has an endpoint in common with the shorter side of the original triangle, as in Figure 24. Then in terms of the notation of that figure

$$\frac{\sin \alpha}{h} = \frac{\sin 90}{b} = \frac{\sin (90 - \alpha)}{m},$$

so h and m can be found. Since $c = m + n$, $n = c - m$. Hence h and n are now known, and the angle between them is 90°, as in Case (i). It is possible to find a and β by the methods described for vectors. With α and β known, the remaining angle may be found.

• Example:
Given a triangle in which $\alpha = 32.6°$, $b = 537$, $c = 428$, find the remaining parts. From vertex B draw a perpendicular to side AC. Then $90 - 32.6 = 57.4$, and

$$\frac{\sin 32.6°}{h} = \frac{\sin 90°}{428} = \frac{\sin 57.4°}{m}$$

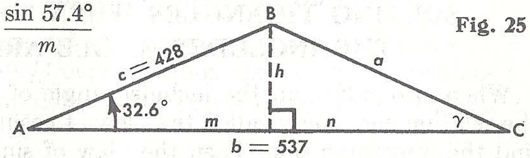


Fig. 25

Set 1 of S over 428 on D. Under 32.6° on S read $h = 231$ on D. Move hairline over 57.4° on S, and read $m = 361$ on D. Now $n = 537 - 361 = 176$. Set the right-hand index of C over 231 on D. Move the hairline to 176 on D. Read $\gamma = 52.7°$ on T. Move the slide so 52.7° on S (reading from right to left) is under the hairline. At the index of C read $a = .290$ on D. The remaining angle can then be found by calculating the value of $180 - (32.6 + 52.7)$, which is 93.7°.

Case (iii). If the included angle is obtuse (that is, greater than 90°), draw a perpendicular from the end of one of the known sides to the other known side *extended.* This perpendicular will be outside the original triangle as in Figure 26. In terms of the notation used in Figure 26, the following relation exists:

$$\frac{\sin (180 - \alpha)}{h} = \frac{\sin 90}{b} = \frac{\sin (\alpha - 90)}{m}.$$

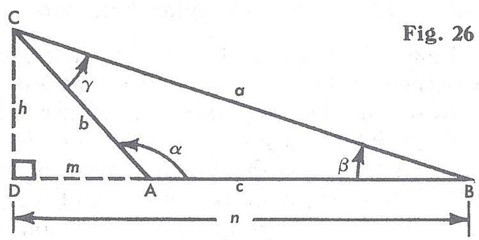


Fig. 26

Therefore h and m can be found. Then $n = m + c$ can also be found, and in right triangle CDB, the two legs are now known. Hence as noted under Case (i), methods described for vectors may be used to find a, β, and finally, γ.

• Example:
Given a triangle in which $\alpha = 110.5°$, $b = 62.4$, and $c = 23.5$, find the remaining parts (See Figure 27). Since $180 - 110.5° = 69.5°$,

$$\frac{\sin 69.5}{h} = \frac{\sin 90}{62.4} = \frac{\sin 20.5.}{m}$$

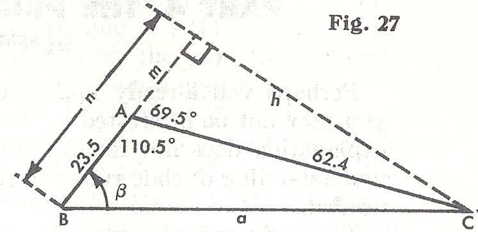


Fig. 27

Set $\sin 90° = 1$ of S over 62.4 on D. Move hairline to 69.5° on S, read $h = 58.4$ on D. Move hairline to 20.5° on S, and read $m = 21.8$ on D. Now $n = 21.8 + 23.5 = 45.3$. Set the index of C over 45.3 on DI. Move the hairline to 58.4 on DI, and read $\beta = 52.2°$ on T. Move the hairline to 52.2 on S (reading from right to left), and read $a = 74.0$ on DI. The remaining angle is equal to $180 - (110.5 + 52.2)$ or 17.3°.

PROBLEM SET XXVI

Find the missing parts when the data are as follows.

1. $b = 56.2$ $c = 63.9$ $\alpha = 71.5°$	**2.** $a = 302$ $b = 427$ $\gamma = 134.5°$
3. $a = 3.57$ $c = 2.92$ $\beta = 31.1°$	**4.** $b = 0.431$ $c = 0.890$ $\alpha = 29.9°$
5. $b = 7260$ $c = 9380$ $\alpha = 78.8°$	**6.** $b = 0.657$ $c = 0.319$ $\alpha = 108°$
7. $a = 0.983$ $c = 1.257$ $\beta = 57.6°$	**8.** $a = 21.5$ $b = 13.2$ $\gamma = 19.3°$

SOLVING TRIANGLES WHEN THREE SIDES ARE KNOWN

When three sides of a triangle are known, one of the angles may be found by using the law of cosines. Thus when a, b, and c are known the relation $a^2 = b^2 + c^2 - 2\,bc \cos \alpha$ may be solved for $\cos \alpha$, and from this the value of α may be found. Then the law of sines may be used to find the other angles.

• Example:
Given $a = 10$, $b = 12$, and $c = 14$, find α. Write $10^2 = 12^2 + 14^2 - 2 \times 12 \times 14 \cos \alpha$, and solve for α. Then

$$\cos \alpha = \frac{12^2 + 14^2 - 10^2}{2 \times 12 \times 14}, \text{ or } \cos \alpha = \frac{144 + 196 - 100}{336} = \frac{240}{336}.$$

Set the right index of C over 336 on D. Move the indicator to 240 on D. Read S right-to-left to find $\alpha = 44.4°$.

PART 4. THE PRINCIPLES UNDERLYING SLIDE RULES

Perhaps you already understand how a slide rule "works". If so, you may not be interested in the simple explanation that follows. This explanation does not require any knowledge of *logarithms*. For a full understanding of slide rules, however, some knowledge of logarithms is needed.

The ordinary slide rule is used to multiply and divide numbers, and to do many other more complicated computations. You can understand better how it works if you take a few minutes to see how a slide rule for addition can be made.

SCALES FOR ADDITION

You have used a foot-rule many times to measure lengths. Two foot-rules used together can be used to add numbers.

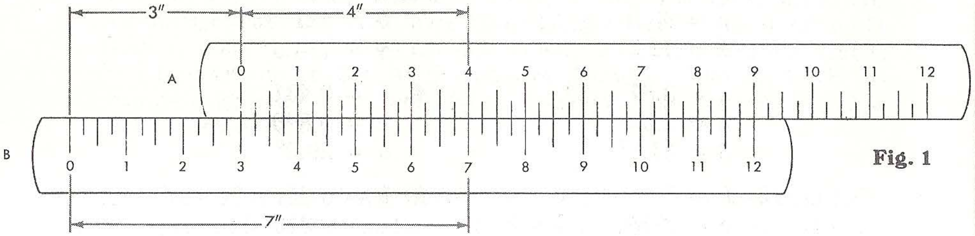
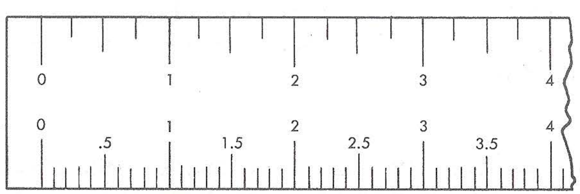


Fig. 1

For example, in Figure 1 the 0 of Rule A is placed opposite 3 of Rule B. Under 4 of Rule A you see 7 of Rule B. Used in this way, these rules provide a mechanical way of showing that $3 + 4 = 7$. At the same time, Figure 5 shows that $3 + 5 = 8$, and other sums. Also, by sliding Rule A along Rule B, the 0 mark of Rule A can be placed opposite any mark on Rule B. The two rules used together become a *slide rule* for simple addition.

The marks (or "graduations") and numerals on a foot-rule are an example of a *scale*.

Fig. 2



The scale on a foot-rule reads from 0 to 12 because the rule is 12 inches long.

Foot-rules are not often used to add as in Figure 1 because the scales are short. Only the numbers from 0 to 12 are represented. You remember

facts such $3 + 4 = 7$, and do not need a mechanical device to do such simple examples. Also notice that the example $3 + 11$ causes trouble because the numeral 11 on the A-scale is on the part that extends beyond the B-scale. There is no numeral 14 on a foot-rule.

Examples such as $3 + 11 = 14$ and $18 + 17 = 35$ can be done mechanically by using two yardsticks. The scale on a yardstick reads from 0 to 36. However, an example such as $18 + 27$ causes trouble because the sum, 45, is not on the scale of a yardstick. By using two longer scales this example can be done. In fact, *any* addition example can be done *by using scales that are long enough,* but very long scales would not be convenient to carry around or use. To be convenient, a slide rule scale should not be much more than about 10 inches in length.

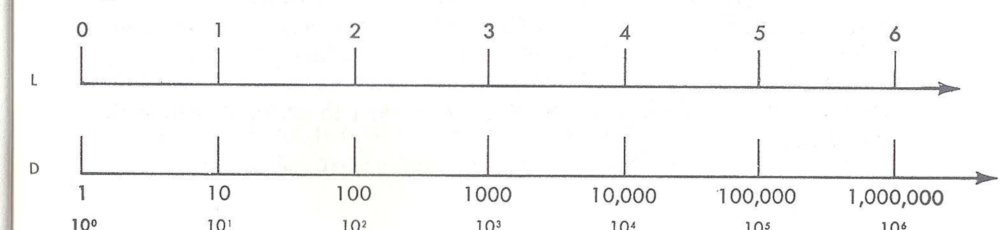
The scale on a foot-rule has marks to represent fractions. Usually the marks show *eighths*, but sometimes *sixteenths* and even finer subdivisions are made. By using these marks, examples such as $2\frac{3}{8} + 5\frac{5}{16} = 8\frac{11}{16}$ can be done easily with sliding foot-rules. Marks to represent decimal fractions, such as *tenths*, can also be used, as shown on the bottom scale of Figure 2. By using scales graduated in tenths, examples such as $2.3 + 6.9 = 9.2$ can easily be done. You can see, however, that graduations to show *hundredths* would be too close together to read without a magnifier.

In the pages that follow you will learn how the ordinary slide rule overcomes the kinds of difficulties mentioned above. The difficulties arise because the scales must be short. If the scales could be as long as we pleased some things would be easier, but as a mechanical device the slide rule would not be the very convenient tool it now is.

SCALES FOR MULTIPLICATION

The scale of a foot-rule is uniform. The numerals 0, 1, 2, 3, etc. represent consecutive integers (sometimes called whole numbers). On a foot-rule, the distance between two graduations which represent consecutive integers is always one inch (see scale L of Figure 3). You can make a *non-uniform* scale by labelling the graduations as is done in scale D of Figure 3. The graduation labelled 0 on L is labelled 1 on D; the mark labelled 1 on L is labelled 10 on D; the mark labelled 2 on L is labelled

Fig. 3



100 on D; etc. The marks for 2, 3, 4, etc. to 9 which would be between 1 and 10 on scale D are not shown in Figure 3. The sub-division marks

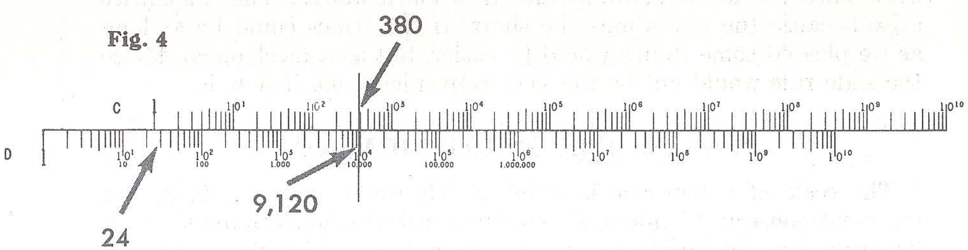
for 11, 12, 13, etc. to 99 which would be between 10 and 100 are not shown either. These 89 sub-division marks would have to be put in the same length (1 inch) as the 8 marks between 1 and 10. They would, therefore, be much closer together than they are on scale L.

Similarly the 899 sub-division marks for integers between 100 and 1000 are not shown in Figure 3. These marks would be very much closer together than those with the same numerals would be on scale L. The graduations get more and more crowded as the numbers get larger. Scale D is a *non-uniform scale,* and this particular non-uniform scale is called a logarithmic scale.

You should notice that these scales may be extended indefinitely. The only thing that limits the range of the numbers is the amount of paper or other material you can use to put them on. If you had a strip of paper a mile long, the numerals in scale L would range from 0 to 63,360. The numerals on scale D would then range from 1 to an enormous number. This number would have 63,361 digits or figures, and would require a strip of paper about 440 feet long on which to type them side by side. Some idea of the size of this number is gained by remembering that it requires only ten digits to write "one billion" or 1,000,000,000.

A slide rule which has two logarithmic scales (like scale D of Figure 3) can be used to multiply two numbers. For example, suppose you

Fig. 4



want to multiply 24 × 380. You would place the mark for 1 on one of the scales (labelled C in Figure 4) over the mark for 24 on the other scale. Then you would look for the mark 380 on scale C. On scale D right under it you would read your answer 9,120.

Two logarithmic scales, if used just as the foot-rules were for addition, will provide the answers for multiplication examples. However, to be practical they must be short—only about 10 inches or less in length. Also, many of the subdivision graduations (such as for 24 or 380 in Figure 4) cannot be shown, so the user must learn to estimate where they would be.

HOW SHORT SCALES CAN BE USED

If you want to measure a distance of several feet, but have only a foot-rule, you measure by using the same scale over and over again. That is, you put the scale down and make a mark to show the first 12 inches, then slide the scale along and use it again. You repeat this as many times as necessary. In this way you can measure as much length as you wish with only a foot-rule but *you must count and keep track or remember how many times the scale was used.*

Similarly, if you want to multiply any two numbers by using logarithmic scales, you can do it with scales that show only the numerals from 1 to 10, but you must then keep track of the decimal point by some other scheme. In reality, you use the same single section or piece of scale over and over again.

For example, consider again the problem of multiplying 24 × 380 by using the C and D scales of an ordinary slide rule. First, the left hand of the C scale is set over 24 on the D scale. In this case the D scale represents the section between 10 and 100 of the complete logarithmic scale. The numeral 2 on the scale represents 20 and the numeral 3 represents 30. The fourth main subdivision between 2 and 3 represents 24. Second, the hairline of the cursor is set over 380 on the C scale. In this case the C scale represents the section between 100 and 1000 of the complete logarithmic scale.

When the setting is made and read in this way it is the same as putting down the C scale once and saying "ten", then putting it down again, and saying "one hundred", and then going on to 380. You are now in the section for "hundreds"; the numeral 3 represents 300 and the numeral 4 represents 400. The eighth main sub-division between 3 and 4 represents 380 (see Figure 4).

Now the hairline of the cursor is just to the right of the numeral 9 of the D scale. Since you started at 24 of the D scale, you might think this should represent 90. However, because two lengths of C scale were used in reaching the "hundreds", these two lengths must also be accounted for on the D scale. The numeral 9 near the hairline is now read as 9000 (see Figure 4). This example shows how a single section of the logarithmic scales can be used to get the answers if you know how to interpret the numerals.

In using a slide rule for computation the decimal point in the answer is always found mentally or by some auxiliary method. It is not possible to construct scales which have enough sections to care for all numbers by purely mechanical methods. By shortening each section, however, it is possible to construct scales of convenient length which will provide for the range 0.000,000,000, 1 to 10,000,000,000 *with the decimal point shown.* Such scales are called *"Decimal Keeper"* scales.[1]

[1]Slide rules with these scales, which may be used to locate the decimal point, or to teach slide principles, are available from Pickett & Eckel, Inc.

HOW LOGARITHMIC SCALES ARE MADE

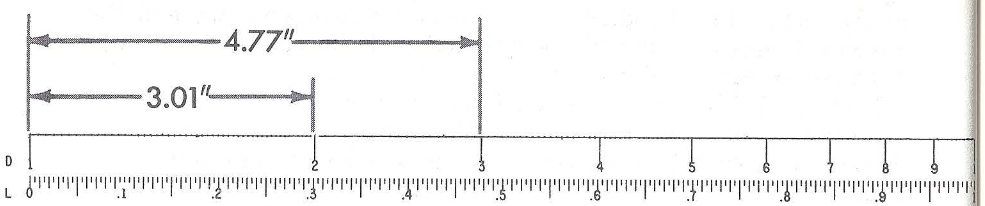
Let n denote any number except zero. Then it is possible to find another number, x, such that $10^x = n$. When defined in this way, the number x is called "the logarithm of n to the base 10", and it is customary to abbreviate this by writing $x = \log_{10} n$. For example, $10^{0.301} = 2$, so $0.301 = \log_{10} 2$. Tables of logarithms are often used to find the logarithm of any specified number.

When a slide rule D scale is to be made, the formula $d = M \log_{10} n$ is used. In this formula the number M determines the length of the scale. If the unit is the centimeter, usually M = 25, approximately, or about 10 inches. For convenience, M = 10 will be used in the following discussion. The following pairs of numbers are easily found by using the formula $d = 10 \log n$ and a table of logarithms.

n	1	2	3	4	5	6	7	8	9	10
d	0	3.01	4.77	6.02	6.99	7.78	8.45	9.03	9.54	10.00

Now on a segment of straight line 10 inches long, it is easy to measure a distance of 3.01 inches from the left end and put down a graduation mark. Then the numeral 2 is placed beside this mark. Next, a distance

Fig. 5



of 4.77 inches is measured, and the graduation mark is labeled 3, etc. In this way the primary graduations of a D scale can be placed.

The same procedure is used to locate the secondary and tertiary graduations. For example, if $n = 1.3$, the table of logarithms shows that $\log 1.3 = 0.114$; hence $d = 10 \times (0.114) = 1.14$ inches.

Similarly, there is a scale formula for each of the different scales. For the A scale, the formula is $d = (M/2) \log_{10} n$. For the S scale the formula is $d = M \log (10 \sin n)$, and for the CI scale, the formula is $d = M \log (10/n)$.

The number of different scales that can be put on a rule, their length, and the number of subdivisions to be put on at various places on the scales are largely determined by the amount of space available.

THE PRINCIPLE OF THE SLIDE RULE EXPLAINED IN TERMS OF LOGARITHMS

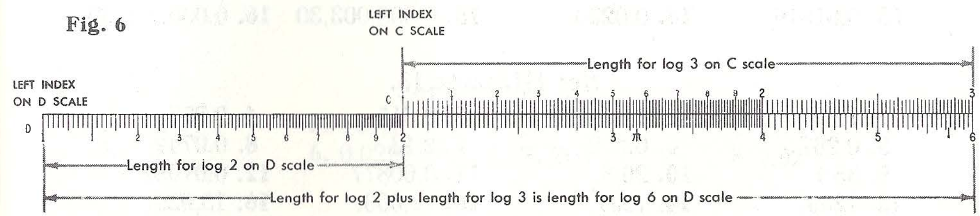
In the theory of logarithms it is proved that the logarithm of a product is equal to the sum of the logarithms of the factors. In symbols, if $n\ m = p$ then

$$\log n + \log m = \log p.$$

For example, $2 \times 3 = 6$, and $\log 2 + \log 3 = \log 6$.

As explained in the preceding section, the mark indicated by the numeral 2 on the D scale is placed at a distance from the left index of

Fig. 6



the D scale determined by log 2. Similarly, the mark beside 3 on the C scale is placed at a distance determined by log 3. The sum of these lengths therefore represents log 6. On the D scale, the numeral 6 is opposite the mark determined by log 6, so when the scales are set as in Figure 6, the product can be read on the D scale.

The logarithm of the quotient of two numbers is equal to the logarithm of the dividend minus the logarithm of the divisor. Hence quotients are found by subtracting lengths of scale determined by the logarithms of the numbers involved.

The scale formula for the D scale is $d_D = M \log n_D$ and for the A scale it is $d_A = (M/2) \log n_A$. When the hairline is set at any distance d_D from the left index of the D scale, it is automatically set at the same distance from the left index of the A scale; that is, $d_D = d_A$.

When this is true

$$M \log n_D = (M/2) \log n_A,$$

and

$$\log n_D = \log n_A^{\frac{1}{2}}.$$

Hence $n_D = \sqrt{n_A}$, or $n_A = n_D^2$.

Therefore the graduation on the D scale for any number is opposite the graduation on the A scale for the square root of the number, and conversely.

A similar analysis will show the relationship of each scale on a slide rule to every other scale, but some of the relations are rarely used. All of the most frequently used relations have been explained in Part 1 and Part 2 of this manual without depending upon a knowledge of logarithms or referring to the scale formulas for the various scales.

ANSWERS.

Set I, page 9.

1. 55.5	**2.** 79.8	**3.** 242.	**4.** 159.3
5. 5.81	**6.** 8360.	**7.** 80.9	**8.** 361.
9. 123.3	**10.** 68.4	**11.** 77.5	**12.** 9.88
13. 74.7	**14.** 92,300.	**15.** .0899	**16.** 0.0815

Set II, page 12.

1. 326.	**2.** 617.	**3.** 634,000.	**4.** 547,000.
5. 10.88	**6.** 7.87	**7.** 0.249	**8.** 0.272
9. 0.001161	**10.** 0.001596	**11.** 1.264	**12.** 1.245
13. 0.01916	**14.** 0.0226	**15.** 0.000,003,30	**16.** 0.000,003,30

Set III, page 13.

1. 1.62	**2.** 11.86	**3.** 0.815	**4.** 0.873
5. 0.267	**6.** 0.268	**7.** 2.13	**8.** 0.0749
9. 35.1	**10.** 26.8	**11.** 0.00877	**12.** 0.01082
13. 1259.	**14.** 1531.	**15.** 12,000.	**16.** 15,920.
17. 391,000	**18.** 385,000	**19.** 0.000,044,5	**20.** 0.000,055,9

Set IV, page 14.

1. 73.9	**2.** 6,840	**3.** 167	**4.** 16.24
5. 5260			

Set V, page 15.

1. 61.1	**2.** 2.07	**3.** 53.7	**4.** 1.081
5. 351	**6.** 97.0	**7.** 104.8	**8.** 2930
9. 0.0000147 or 1.47×10^{-5}		**10.** 11,190	

Set VI, page 17.

1. 1.25	**2.** 42.5	**3.** 13.4	**4.** 65.7
5. 300	**6.** 40.7	**7.** 1247	**8.** 1136
9. 276	**10.** 26.4	**11.** $a=22.7$; $b=36.8$	
12. $x=23.2$; $y=33.6$	**13.** $a=3.55$; $b=82.9$	**14.** $r=135.$; $s=7.59$	
15. $x=0.435$; $y=102.6$	**16.** $a=1.6$; $b=720$; $c=4.23$		

Set VII, page 18.

1. 55.3	**2.** 54.4	**3.** 120	**4.** 1391
5. 130	**6.** 86.1	**7.** 1006	**8.** 69200
9. 4.12	**10.** 14.96	**11.** .292	**12.** 0.360
13. 145	**14.** 198.8	**15.** 1589	**16.** 1287.
17. 2.04	**18.** 2.08	**19.** 32.1	**20.** 19.00
21. 3.79	**22.** 4.65	**23.** 5.17	**24.** 4.97
25. 659	**26.** 346	**27.** 3.06	**28.** 4.06
29. 0.168	**30.** 1.345		

Set VIII, page 18.

1. 8.16%.	**2.** $64.50	**3.** $213.50	**4.** 925 ft./sec.
5. About 2.5 days	**6.** 108 mi./hr.	**7.** 1.5 amp.	

Set IX, page 21.

1. 0.143	**2.** 0.0284	**3.** 5.57	**4.** 0.000,155,5
5. 41.7	**6.** 240	**7.** 121.3	**8.** 157.1
9. 13.36	**10.** 9.66	**11.** 2.23	**12.** 1.561
13. 42.7	**14.** 3.16	**15.** 0.455	**16.** 0.379
17. 6,840	**18.** 16.24	**19.** 0.654	**20.** 1.067

Set X, page 23.

1. 11.02	**2.** 123.3	**3.** 17.43	**4.** 8.33
5. 4.20	**6.** 0.0291	**7.** 266	**8.** 9.92

Set XI, page 24.

1. 0.740	**2.** 0.1592	**3.** 2.91	**4.** 0.00653
5. 4.01	**6.** 0.481		

Set XII, page 26.

1. 2.7	**2.** 8.54	**3.** 29	**4.** 6.71
5. 21.2	**6.** 3.16	**7.** 1.28	**8.** 4.05
9. 0.29	**10.** 3.65	**11.** 365	**12.** 15.6
13. 32.9	**14.** 3290	**15.** 9.9	**16.** 26.6
17. 0.266	**18.** 1.77	**19.** 1.58	**20.** 54.8
21. 0.0775	**22.** 2320	**23.** 0.00757	**24.** 0.000,008,07
25. 9.6	**26.** 0.249	**27.** 0.00097	**28.** 1.93
29. 5.15	**30.** 8.77	**31.** 2.76	**32.** 78.3
33. 0.1024	**34.** 6.92	**35.** 2290.	**36.** 0.00518

Set XIII, page 29.

1. 1.59	**2.** 3.42	**3.** 7.36	**4.** 0.159
5. 0.342	**6.** 0.736	**7.** 4096	**8.** 4.096
9. 0.004,096	**10.** 4910	**11.** 14.7	**12.** 0.402
13. 4,450,000	**14.** 176,600	**15.** 31.	**16.** 1.754
17. 4.14	**18.** 9.34	**19.** 0.316	**20.** 80.7
21. 0.538	**22.** 0.81	**23.** 0.238	**24.** 0.000,009,2
25. 0.104	**26.** 0.003375	**27.** 0.000,572	**28.** 0.08
29. 3.76	**30.** 1.465	**31.** 15.5	**32.** 44
33. 91			

Set XIV, page 31.

1. 13.42 **2.** 10.58 **3.** 70.8 **4.** 41.9
5. 42.3 **6.** 19.22 **7.** 12,250. **8.** 32.8
9. 2.92 **10.** 2.27 **11.** 241. **12.** 38.1
13. 11.31 **14.** 70.8 **15.** 190. **16.** 54.0
17. 0.1476 **18.** 1.620 **19.** 0.947 **20.** 0.0335
21. 7.02 **22.** 1.88 **23.** 3.69 **24.** 12.64

Set XV, page 31.

1. 12.48 **2.** 7.94 **3.** 42.7 **4.** 45.8
5. 1.747 **6.** 7.47 **7.** 1.083 **8.** 0.983
9. 11,800 **10.** 650. **11.** 1510. **12.** 132.2
13. 6.26 **14.** 160. **15.** 4.68 **16.** 1.458
17. 13.08 **18.** 1.819 **19.** 12.57 **20.** 1.769
21. 0.1939 **22.** 4.03 **23.** 8.86

Set XVI, page 32.

1. 8.8 sq. ft. **2.** 576 cu. ft. **3.** 53 cu. ft. **4.** 209 ft.
5. 1160 ft. **6.** \$2.69 **7.** 5.64 in. **8.** 34.6 in.
9. 18.3, 22.6, 15.0, 8.6, 12.9, 9.7, 7.5, 5.4. **10.** \$15.30 **11.** 47.8 ohms.

Set XVII, page 34.

1. 2.738 **2.** 7.093-10 **3.** 4.358 **4.** 9.831-10
5. 0.513 **6.** 8.288-10 **7.** 1.598 **8.** 7.146-10
9. 4 **10.** 5.000-10 **11.** 500 **12.** 8.24
13. 5570 **14.** 0.00427 **15.** 668 **16.** 0.2275
17. 0.0194 **18.** 735 **19.** 112.7 **20.** 0.678

Set XVIII, page 37.

Problem number	x	sin x	cos x	tan x	cot x	sec x	csc x
1.	**16.5**	**0.284**	**0.959**	**0.296**	**3.38**	**1.043**	**3.52**
2.	**20**	0.342	0.940	0.364	2.75	1.064	2.92
3.	60°	**0.866**	0.500	1.732	0.577	2.000	1.155
4.	**42°**	0.669	0.743	0.900	1.111	1.346	1.495
5.	73.4°	0.958	**0.286**	3.35	0.298	3.500	1.044
6.	**68°**	0.927	0.375	2.48	0.404	2.67	1.079
7.	**11.72°**	0.203	0.979	0.207	4.82	1.021	4.93
8.	18.2°	**0.312**	0.950	0.329	3.04	1.053	3.20
9.	26.7°	0.449	**0.893**	0.503	1.99	1.120	2.23
10.	39.4°	**0.635**	0.773	0.821	1.217	1.294	1.575
11.	62.9	0.890	0.456	1.954	0.512	2.193	**1.124**
12.	81.5	0.989	0.1478	6.69	0.1495	**6.76**	1.011

Set XIX, page 38.

1. 18.6 **2.** 66.4° **3.** 31.7° **4.** 83.85
5. 49.55° **6.** 68.84° **7.** 9.6° **8.** 37.2
9. 10° **10.** 78.5° **11.** 28.2 **12.** 10.8°
13. 71.5 **14.** 76. **15.** 12.2 **16.** 28.6°
17. 37.4 **18.** 9.32° **19.** 53.3 **20.** 22.9
21. 47.7 **22.** 46.7 **23.** 6.06 **24.** 6.33
25. 10.3° **26.** 79.7°

Set XX, page 40.

1. 0.0244 **2.** 0.0489 **3.** 0.1431 **4.** 73.5
5. 0.0635 **6.** 0.00070 **7.** 14.6 **8.** 0.00061
9. 0.99978 **10.** 38.2 **11.** 1.3° **12.** 2.32°
13. 3.1° **14.** 2.5° **15.** 0.71° **16.** 0.93°
17. 1.6° **18.** 0.745° **19.** 5.1° **20.** 4.51°

Set XXI, page 41.

1. 6.51, 1.207, 17,810, 0.312, 67.2
2. 4.87, 17.48, 1.089, 66.4, 10.27
3. 17.2, 68.8, 150, 447, 26.4, 47.0, 99.1, 339., 172, 2.29, 75, 34.6, 22.5, 270, 144
4. 0.1396, 0.698, 1.326, 2.65, 4.56, 5.71, 0.0401, 0.0908, 0.225, 1.038, 1.92, 3.32, 3.53, 5.375, 6.11
5. 0.000669, 0.00189, 0.001401, 0.00817, 0.01134. These answers are also the sine and the tangent.
6. 0.00001746, 0.0000873, 0.0001212, 0.0001852, 0.000272

Set XXII, page 42.

1. 5.38 **2.** 122.7 **3.** 1.924 **4.** 77.3
5. 48.2 **6.** 10.03 **7.** 31.6 **8.** 26.4
9. 0.278 **10.** 0.928 **11.** 111. **12.** 40.7
13. 1.438 **14.** 0.00974 **15.** 0.00813 **16.** 15.82
17. 2.43 **18.** 15.78 **19.** 567. **20.** 0.778

Set XXIII, page 45.

1. $\gamma=87°$, $b=69.5$, $c=76.6$
2. $\beta=64.6$, $a=20.7$, $b=43.6$
3. $\alpha=30°$, $a=56.1$, $b=38.4$
4. $\gamma=54°$, $b=10.53$, $c=8.96$
5. $\alpha=77.4°$, $a=82.3$, $c=84.3$
6. $\alpha=43.1°$, $\beta=46.9°$, $b=5.57$
7. $\gamma=14.1°$, $\beta=147.7$, $b=119$
8. $\alpha=20°$, $\gamma=80°$, $a=12.99$
9. $\beta=16.5°$ or $163.5°$, $\gamma=153°$ or $6.0°$, $c=39.9$ or 9.2
10. $\gamma=102$, $a=52.8$, $b=833.$
11. $\beta=85°$, $b=7.09$, $c=7.11$
12. $\gamma=90°$, $\beta=71°$, $b=79.5$

13. $\beta=47.4^\circ$ $a=969$ $b=1054$	**14.** $\alpha=30^\circ$ $\beta=60^\circ$ $b=0.416$	**15.** $\gamma=62.8^\circ$ or 117.2° $\beta=97.2$ or 42.8° $b=174$ or 119.2	**16.** $\beta=2^\circ$ $b=3.54$ $c=7.08$
17. $\alpha=3.72^\circ$ $\beta=86.28$ $b=24.7$	**18.** $\alpha=45^\circ$ $a=253$ $b=253$	**19.** $\alpha=0.143^\circ$ $\beta=89.857^\circ$ $b=800$	**20.** $\beta=40.8^\circ$ $\gamma=69.2^\circ$ $c=0.0458$
21. $\alpha=59.8^\circ$ or 120.2° $\gamma=69.6^\circ$ or 9.2° $c=92.5$ or 15.8			

Set XXIV, page 49.

1. 36.9° 53.1° 35	**2.** 39.8° 50.2° 78.1	**3.** 9.46° 80.54 243	**4.** 32.3° 57.7° 23.2
5. 68.7° 21.3° 84.0	**6.** 76.5° 13.5 2480	**7.** 43.1° 46.9° 54.9	**8.** 63.8° 26.2 0.469
9. 46.2° 43.8° 5280	**10.** 5.43° 84.57° 988	**11.** 90.7 /58.1°	**12.** 1.101 /35.8°
13. 0.650 /82.1°	**14.** 16.2 /0.88°	**15.** 0.00775 /29.8°	**16.** 157 /50.7°
17. 6.79 /151.9°	**18.** 21.8 /232.1°		

Set XXV, page 51.

1. $24.7+j8.04$	**2.** $20.4+j5.74$	**3.** $3.26+j7.69$
4. $0.479+j0.0377$	**5.** $282+j278$	**6.** $6.84+j32.2$

Set XXVI, page 53.

1. $\beta=49.1^\circ$ $\gamma=59.6^\circ$ $a=70.5$	**2.** $\beta=26.8^\circ$ $\alpha=18.7^\circ$ $c=674.$	**3.** $\alpha=94.2^\circ$ $\gamma=54.7^\circ$ $b=1.85$	**4.** $\beta=22.6^\circ$ $\gamma=127.5^\circ$ $a=0.559$
5. $\beta=41.8^\circ$ $\gamma=59.4^\circ$ $a=10{,}690.$	**6.** $\beta=50.1$ $\gamma=21.9$ $a=0.814$	**7.** $\alpha=48.6^\circ$ $\gamma=73.8$ $b=1.101$	**8.** $\beta=25.7^\circ$ $\alpha=135^\circ$ $c=10.10$

HOW TO ADJUST YOUR SLIDE RULE

Each rule is accurately adjusted before it leaves the factory. However, handling during shipment, dropping the rule, or a series of jars may loosen the adjusting screws and throw the scales out of alignment. Follow these simple directions for slide rule adjustment.

CURSOR WINDOW HAIRLINE ADJUSTMENT

Line up the hairline on one side of the rule at a time.

1. Lay rule on flat surface and loosen adjusting screws in end plates.
2. Line up C index with D index. Then align DF (or A) index with CF (or B) index.
3. Tighten screws in end plates.
4. Loosen cursor window screws. Slip a narrow strip of thin cardboard (or 3 or 4 narrow strips of paper) under center of window.
5. Align hairline with D and DF (or D and A) indices, and tighten cursor window screws. Check to see that *window surfaces do not touch or rub against rule surfaces.*

Note: The narrow strip of cardboard under the window will prevent possible distortion or "bowing in" of the window when screws are tightened. "Bowing in" may cause rubbing of window against rule surface with resultant wear or scratches.

Line up hairline on reverse side of rule.

1. Loosen all 4 cursor window screws.
2. Place narrow strip of thin cardboard under window to prevent "Bowing in" when screws are tightened.
3. Align hairline and indices on first side of rule, then turn rule over carefully to avoid moving cursor.
4. Align hairline with indices and tighten cursor screws.
5. Check to see that window surfaces do not touch surfaces of rule during operation.

SLIDER TENSION ADJUSTMENT • Loosen adjustment screws on end brackets; regulate tension of slider, tighten the screws using care not to misalign the scales. The adjustment needed may be a fraction of a thousandth of an inch, and several tries may be necessary to get perfect slider action.

SCALE LINE-UP ADJUSTMENTS • (1) Move slider until indices of C and D scales coincide. (2) Move cursor to one end. (3) Place rule on flat surface with face uppermost. (4) Loosen end plate adjusting screw slightly. (5) Adjust upper portion of rule until graduations on DF scale coincide with corresponding graduations on CF scale. (6) Tighten screws in end plates.

REPLACEABLE ADJUSTING SCREWS • All Pickett All-Metal rules are equipped with Telescopic Adjusting Screws. In adjusting your rule, if you should strip the threads on one of the Adjusting Screws, simply "push out" the female portion of the screw and replace with a new screw obtainable from your dealer, or from the factory. We do not recommend replacing only the male or female portion of the screw.

HOW TO KEEP YOUR SLIDE RULE IN CONDITION

OPERATION • Always hold your rule between thumb and forefinger at the ENDS of the rule. This will insure free, smooth movement of the slider. Holding your rule at the center tends to bind the slider and hinder its free movement.

CLEANING • Wash surface of the rule with a non-abrasive soap and water when cleaning the scales.

LUBRICATION • The metal edges of your slide rule will require lubrication from time to time. To lubricate, put a *little* white petroleum jelly (White Vaseline) on the edges and move the slider back and forth several times. Wipe off any excess lubricant. *Do not use ordinary oil as it may eventually discolor rule surfaces.*

LEATHER CASE CARE • Your Leather Slide Rule Case is made of the finest top-grain, genuine California Saddle Leather. This leather is slow-tanned using the natural tanbark from the rare Lithocarpus Oak which grows only in California. It polishes more and more with use and age.

To clean your case and to keep the leather pliable and in perfect condition, rub in a good harness soap such as Propert's Harness Soap.

PICKETT, INC.

SANTA BARBARA, CALIF.