INSTRUCTIONS

FOR THE USE OF

FOWLER

"Magnum" Long Scale

CALCULATOR

FOWLER'S (CALCULATORS) LTD
Hampson Street Works
SALE, MANCHESTER

A few of the Professions and Trades for which Fowler's Calculators are especially suitable

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A

BRITISH PRECISION MADE INSTRUMENT

BACKED BY FIFTY YEARS' EXPERIENCE

FOWLER'S "MAGNUM" CALCULATOR.

Fowler's "Magnum" Calculator, like the well-known waistcoat pocket instrument consists of a series of concentric circular scales, logarithmically divided and mounted on a dial capable of rotation by a thumb nut outside the containing case The scales are equipped with a fixed radial datum line, and a radial cursor line rotated by a second thumb nut. The rotating scales cursor line, and operating mechanism are enclosed in an airtight electro-plated metal containing case, fitted with a glass face so that the scales are always kept clean and the instrument is preserved from external injury in a handsome leather wallet which fits easily into a side pocket. The large size of the "Magnum" enables all the scales to be mounted on one dial, and so of being synchronised and read concurrently. It also permits of the use of larger figures and easier reading, an advantage to persons of weak eyesight. Another important feature is that the scales are longer and admit of finer graduation and more accurate reading. This is specially noticeable in the "Long-Scale" which gives a length of 50 ins. as compared with 10 ins. in the ordinary slide-rule and permits of calculations being made to four, and sometimes five, significant figures. The motions of the scales and cursor being effected by gearing, adjustments can be made with great ease and nicety. There is none of the objectiobable sticking or slackness often so troublesome with the straight, slide rule owing to changes of climate or temperature, nor is there any of the "end switching" often necessary with that instrument when working with the full length scale and which induces many users to work habitually with the half length. In circular calculators the scales are continuous and there is no half length.

To sum up the merits of Fowler's "Long-Scale" Instruments, they are more portable, comprehensive and accurate; cleaner and easier to operate, and may be used equally well in any climate.

Logarithmic Calculation.—The mathematical basis on which all instruments of this kind rest is that of logarithms, first discovered by Napier, which permit of the tedious arithmetical operations of multiplication and division being replaced by the simpler ones of addition and subtraction, which can be done with two sliding scales having logarithmic distances marked on them as in the straight slide-rule.

Logarithmic graduation, however, is not uniform and such scales, whether straight or cicular, have certain drawbacks. The intervals which represent logarithmic distances between the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, diminish rapidly from 1 to 10. If the intervals be expressed as percentages of the whole length they are respectively 30, 17·6, 12·5, 9·7, 7·9, 6·7, 5·8, 5·1, 4·5, and show how much finer the interval at the beginning between 1 and 2 can be graduated than the one at the end between 9 and 10. One is nearly 7 times as great as the other and it is impossible in a straight scale of reasonable length to secure uniform graduation. Some parts are divided by 10, others by 5 and others again by only 2. This dissimilarity of division prevents equal accuracy of reading in all parts of the scale.

A length of less than 10 inches in a straight slide is not of much practical use and is not a very portable article. If the scale, however, is arranged in circular form, round one circle or round several, as in Fowler's "Long-Scale," it may afford a length of 30 ins. and yet be capable of insertion in the waistcoat pocket, while in the case of the "Magnum," which can be carried in the side pocket, the length of the Long Scale is 50 inches. The advantage of this is shown in the following examples.

Description of Scales.—There are seven separate scales. Beginning with largest in diameter, No. 1, and proceeding inwards to the smallest, they are as follows :—

Scale No. 1.—The "Short-Scale," a single circle, 13⅓ inches in circumference, graduated clockwise. This is the calculating scale for multiplication, division, etc., analogous to the ordinary slide rule. It is also used for direct reading and incorporating values of functions on other scales by aid of the cursor or datum line (the cursor by preference, as it is closer to the scales and eliminates parallax).

Between the prime numbers **1** and **2** the scale is divided into 20 figured parts (11, 12, 13, ...) each decimally graduated and capable of further graduation with the cursor. Readings on this part can be made **easily to four,** and sometimes five, **significant figures.**

Between the prime numbers, **2, 3, 4, 5,** each part is divided into 10, viz., 21, 22, ... up to **3,** 31 ...... up to **4, 41, 42,** ...... up to **5,** though only even divisions, **22, 24, 26,** etc., are figured, and owing to diminishing space as the scale advances the graduation is reduced from 10 to 5.

Between the prime numbers, **5, 6, 7, 8, 9, 10,** each part is divided into 2, viz., 55, 65, ... up to 95, and each of these decimally graduated, so that on this, the finest part of Scale No. 1, readings can be made **to three** and sometimes four **significant figures.**

Scale No. 2.—The "Reciprocal Scale," a single circle, exactly like No. 1, but graduated **contra clockwise** so that the readings on one are the reciprocal values of those radially opposite on the other. On this scale the values increase from **right to left** instead of from left to right as in all the other scales. It should be noted that any value on Scale No. 4 (the Long Scale) must be expressed on Scale No. 1 before its reciprocal value can be read on Scale No 2.

Scale No. 3.—The Square Root Scale. A scale extending round the inner and outer circumference of a common circle and giving the square root values of readings on Scale No. 1 which conversely gives the squares of the values on Scale No. 3.

This scale, which has a total length of 22 inches, may, if desired, be also used for multiplying and dividing, though these operations are usually made on No. 1 (The Short Scale), or No. 4 (The Long Scale).

Scale No. 4.—"The Long-Scale," A scale extending round six circles, beginning at the smallest and continuing round the successive circumferences until it completes the sixth, with a total length of 50 inches

This scale gives the sixth root values of Scale No. 1, which conversely gives the sixth power values of Scale No. 4. It also enables cube roots to be read at a single setting. Since if x is a number :

$$\sqrt[3]{x} = \sqrt[6]{x^2}$$

Therefore, if x on Scale No. 3 is set under cursor the reading on Scale No. 1 is x^2 and the reading on Scale No. 4 is

$$\sqrt[6]{x^2} \quad i.e. \quad \sqrt[3]{x}$$

In a converse way the cubes of numbers may be read directly by setting the number x on Scale No. 4 under the cursor and reading x^3 on Scale No. 3.

A mental estimate of the value of the cube root, or of the cube, is, of course, required to determine on which particular circle of scales Nos. 4 or 3

$$\sqrt[3]{x} \text{ or } x^3$$

are to be found. This will be discussed when giving examples of the use of the scales.

The most valuable feature of Scale No. 4 is its great length (50 inches) which permits of multiplication and division with a degree of accuracy beyond the possibility of any straight slide rule. The Scale is used just like No. 1, in setting factors, but requires a little mental consideration like any other logarithmic scale to determine the result and hence the circle on which it is to be read. This is more fully discussed in describibg the practical use of the scale. Its great length enables it to be divided into 100 figured parts, 1, 2, 3, 4, ... up to 100, and each of these to be graduated decimally, which makes setting and reading very simple. Three significant figures of a result can be written without hesitation, even in the most finely graduated part between 99 and 100, while further division can be made with the cursor and over a great part of the scale results can be read to four and sometimes five, significant figures. The superiority of such a scale over that of the 10 inch slide rule will be manifest to those familiar with that instrument which only permits 2 graduations between 99 and 100 as compared with 10 graduations of the Long-Scale of the "Magnum Calculator."

Scale No. 5.—A Scale of Logarithms uniformly graduated from 0·01 to 0·1.

For a given number read on Scale No. 1 the Naperian logarithm is read on Scale No. 5 and conversely for a given Naperian or Common Logarithm read on Scale No. 5, the corresponding number is read on Scale No. 1.

Scale No. 6.—A Scale of Angles giving Natural Sines and Log. Sines, extending round the inner and outer circumferences of a common circle. The inner circle gives angles from 35 minutes to 5 degrees 45 minutes; the outer circle angles from 5 degrees 45 minutes to 90 degrees, graduated as follows:—

From	35 mins.	to 1	degree	at intervals of	1	min.
,,	1 deg.	,, 3	,,	,,	2	,,
,,	3 deg.	,, 8	,,	,,	5	,,
,,	8 deg.	,, 12	,,	,,	10	
,,	12 deg.	,, 30	,,	,,	20	,,
,,	30 deg.	,, 60	,,	,,	1	deg.
,,	60 deg.	,, 70	,,	,,	2	,,

Scale No. 7.—A Scale of Angles giving Natural Tangents and Log. Tangents. The Scale gives angles from 5 degrees 45 minutes to 45 degrees graduated as follows:—

From 5 deg. 45 min.	to 8 deg.	at intervals of	5	min
,, 8 deg.	,, 12 deg.	,,	10	,,
,, 12 deg.	,, 30 deg.	,,	20	,,
,, 30 deg.	,, 45 deg.	,,	30	,,

The Natural values of Sines and Tangents are read on Scale No. 1 and Log. values on Scale No. 5.

The dial, with its seven scales, is rotated by the thumb nut at the top of the instrument, and the Cursor line c by the thumb nut at the side. The fixed datum line is on the cover glass of the instrument. The unity or zero line is common to all the scales.

Scales No. 1 and No. 4 have the following values specially indicated:

$\sqrt{2}$, $\sqrt{8}$, $\log_e 10$, π, G.E. (gravity English), E.H.P. (Electrical Horse Power), $\pi/4$, G.F. (gravity French).

The difficulty experienced by a beginner in using logarithmic scales of any kind for calculation is largely due to the variable nature of their graduations and of the values attached to them not only to the graduation lines themselves bot to the spaces between them and which must be allowed for when setting values and reading results. The spaces between the prime numbers 1, 2, 3, 4, ... 10 differ greatly and the numbers themselves may represent their simple values or any multiple of 10 thereof, while the fine graduations of the scale may be of the nature of 2, 5, or 10. These features are at first confusing, but when the learner is familiar with them he will find difficulties disappear and that the calculations can be made with confidence and accuracy as well as great saving of labour, and his attention should be first directed to this end. As he acquires facility in the use of the scales he will discover many short cuts in manipulating them and need not strictly follow the instructions here given for his guidance in the form of a number of worked out examples which he is recommended to study in detail, remembering that the Calculator is a tool requiring for its efficient use the exervise of a little common sense and mental arithmetic. It is not an adding machine for totalling money fractions, like a bank clerk, but a device for making rapidly and with practical accuracy the innumerable calculations required by designers, engineers, chemists, draughtsmen, and students in the course of their daily work.

Examples of Calculations made with One Setting. —The following are examples of calculations which can be made with Fowler's "Magnum" Calculator by simply setting the cursor line to a value on one scale and reading the corresponding value on another, and are chosen not for their simplicity but for difficulty and teaching purposes.

Exercises with the Reciprocal Scale No. 2.—Ex. 1

Find the decimal equivalent of $\dfrac{1}{6{\cdot}456}$

Set cursor over 6456 on Scale No. 1. Read unde cursor on Scale No. 2, 0·1548.

In setting the cursor to 6456 on Scale 1, we note that between 6 and 7 there are 20 graduations, the reading advancing clockwise, 605, 610, 615, 620, etc. and 6456 is between 64 and 65, its exact position being estimated. Conceive this space to be divided into 100 parts and advance 56 of these parts past 64, i.e., just a little more than half way.

Reading Scale 2 anti-clock we make the value under the cursor as near as may be 1548.

From inspection of the faction its value is obviously between one-sixth and one seventh and without hesitatio write down the decimal value as 0'1548.

Ex. 2: Find decimal equivalent of $\frac{1}{3475}$

Set cursor over 3475 on Scale 1.
Read 2878 on Scale 2.

The fraction is manifestly less than $\frac{1}{3000}$ and expressed decimally will require 3 cyphers after the decimal point, so we write the answer 0'0002878.

In setting 3475 under the cursor we note it falls between the figured graduations 34 and 36 and that between 34 and 35 there are 5 graduations, each advancing 2, thus 342, 344, 346, 348, 35. About half-way between 346 and 348 is 347, and a shade past this is 3475.

Reading scale No. 2 the cursor is just short of the value 288. We estimate it as 2878 and the answer, therefore, as 0'0002878.

Ex. 3: Find the decimal equivalent of $\frac{1}{0{\cdot}0284}$

Set cursor over 284 on Scale 1. (It is the second graduation line past 28 and the spaces count 2 each.)

The reading on Scale 2 is just past the graduation following the 35 mark reading anti-clockwise, and where each space again counts 2. We estimate the reading as 3521.

By inspection the value of the fraction is seen to be more than $\frac{1}{30}$ and we write down the answer as 35·21.

The three preceding examples are good illustrations of the care required in scale reading; in noting the value of the graduations, and whether the advance of the scale is clockwise or anti-clockwise. Scale No. 2. the Reciprocal Scale, it may be noted, is the only one graduated anti-clockwise.

Exercises in Squares and Square Roots.—Scales No. 1 and No. 3.

The numbers on Scale No. 1 are the squares of the numbers on Scale No. 3, which extends round the inner and then the outer circumference of a common circle.

Conversely the numbers on Scale No. 3 are the square roots of the numbers on Scale No. 1.

The learner is advised to read the two scales, together with the aid of the cursor and compare numbers whose squares and square roots can be easily compared mentally so as to familiarise himself with the scales, e.g., 2 and 4, 3 and 9, 5 and 25, 8 and 64, etc., and afterwards try larger numbers.

The following table shows how the values of squares and square roots can be approximated mentally and so located on the scales of the Calculator :—

	Hence for any Number between	The Square Root is between
$10^2 = 100$	1 and 100	1 and 10
$20^2 = 400$	100 ,, 400	10 ,, 20
$30^2 = 900$	400 ,, 900	20 ,, 30
$40^2 = 1600$	900 ,, 1600	30 ,, 40
$50^2 = 2500$	1600 ,, 2500	40 ,, 50
$60^2 = 3600$	2500 ,, 3600	50 ,, 60
Etc.	Etc.	Etc.

If the number of digits in a number is odd the square root is read on the inner circle of Scale No. 3. If even it is read on the outer circle of that scale.

If a number is less than unity the square is less than the number and the square root greater, e.g.,

$(\frac{1}{2})^2 = \frac{1}{4}$ $(0{\cdot}8)^2 = 0{\cdot}64$ $\sqrt{\frac{1}{100}} = \frac{1}{10}$ $\sqrt{0{\cdot}49} = 0{\cdot}7$

Ex. 1 : *Find the square of* 0·7462.

Set cursor to 7462 on Scale 3.

Read Scale 1 under cursor.

We estimate value to be 557, and as the given number is less than unity, its square will be less and we write the answer, 0·557.

The correct value is 0·5568, but the reading on the scale is a close approximation.

Ex. 2 : *Find the square of* 27·52.

Set cursor to 2752 on Scale 3.

By mental estimation as per table the square is between 400 and 900.

Reading Scale 1 we make it 757 5.

Actually it is 7573, a close approximation.

Ex 3: *Find the square root of* 1728.

The number lies between 1600 and 2500, therefore the root lies between 40 and 50.

Set cursor to 1728 on Scale 1.

It is between 17 and 18, which here stand for 1700 and 1800; the exact point is just short of the third graduation line after 17.

Read answer on outer circle of Scale No. 3 between and 42 where each graduation counts 2.

It is just short of the third, which would be 41·6; we call it 41·58.

This is correct to four figures and is a good illustration of the extreme accuracy of the scale.

Ex. 4: *Find the square root of* 0·00378.

As the number is less than unity the root will be larger than the number.

Set cursor over 378 on Scale No. 1. This is the graduation line preceding 380.

Read on Scale No. 3, outer circle, the result between 61 and 62; we estimate it as 6148 and the answer, therefore, as 0'6148, correct to four figures.

If in Ex. 1 and 2 above we had been asked to give the reciprocals of $27{\cdot}52^2$ and $0{\cdot}7482^2$, i.e. the value of $\frac{1}{27{\cdot}52^2}$ and of $\frac{1}{0{\cdot}7462^2}$ they could have been read without further setting by simply reading the answers on Scale No. 2 (The Reciprocal Scale) instead of Scale No. 1 and shows how convenient this scale occasionally may be.

In Ex. 3 and 4, however, the values $\frac{1}{\sqrt{1728}}$ and $\frac{1}{\sqrt{0{\cdot}00378}}$ could not be given until the square roots, viz., 41·58 and 0·06148 had been first transferred to Scale No. 1 as reciprocal values can only be read from Scale No. 1.

Cubes and Cube Roots—Scales Nos. 1, 3, 4.

The third power or cube of a number can be easily obtained by multiplying itself three times $x \times x \times x = x^3$ or by reading x^2 on Scale No. 1 from Scale No. 3 and multiplying that power again thus ($x^2 \times x = x^3$).

The first method is the simpler and probably the better, as the same scale is used each time.

Cube Roots may be evaluated in a simple way at one setting by reading Scales Nos. 1, 3, and 4 in conjunction, from the fact that the cube root of any number is the same as the sixth root of the square of that number thus $\sqrt[3]{x} = \sqrt[6]{x^2}$

Now Scale No. 4 (The Long Scale) extends over six circles and gives the 6th roots of the numbers on Scale No. 1, which are the squares of numbers on Scale No. 3

The particular circle of Scale No. 4 on which a cube root is located can be determined mentally by a consideration of the following :—

		Hence for any Number between			The Cube Roo is between		
$10^3 = 1000$		1	and	1000	1	and	10
$20^3 = 8000$		1000	,,	8000	10	,,	20
$30^3 = 27000$		8000	,,	27000	20	,,	30
$40^3 = 64000$		27000	,,	64000	30	,,	40
$50^3 = 125000$		64000	,,	125000	40	,,	50

It is easy to say mentally the cube of 2, 3, 4, 5, and add three cyphers and so fix the blocks of numbers to which the cube root of any number between 1 and 50 belongs.

Ex. 1 : *Find the cube root of* 34,680 ($\sqrt[3]{34680}$).

Set cursor over 34680 on outer circumference of Scale No. 3. It is situated just a little less than three-quarters of the distance between the 3rd and 4th graduation following 34.

On Scale No. 1 the cursor lies over the square o the given number, but without paying any attention to the square, seek for the cube root on Scale No. 4, between 30 and 40, because the given number is situated between 27,000 and 64,000. A rapid survey shows that the cube root is under the cursor on the 4th circle reckoning from the centre and between 32 and 33. At this part the scale is graduated in tenths and we make the reading, i.e., the answer, 32·62, which is as close an approximation as can be given in four figures.

Ex. 2 : *Find the cube root of* $\dfrac{1}{6844}$ *i.e.* $\sqrt[3]{\dfrac{1}{6844}}$

First find $\sqrt[3]{6844}$ which as the number is between 1,000 and 8,000 must lie between 10 and 20, and proves to be actually 18·98. So what is required is the value of $\dfrac{1}{18{\cdot}98}$

Set cursor over 18·98 on Scale No. 1.

Read on No. 2 (The Reciprocal Scale) the answer under the cursor. This reading is 527.

From inspection of the fraction $\frac{1}{18 \cdot 93}$ we see its value is round about $\frac{1}{20}$ and, therefore, write down:

$$\sqrt[3]{\frac{1}{6844}} = 0{\cdot}05270.$$

Cube roots or any other roots or powers, whole or fractional, may be obtained by means of logarithms and sometimes this method is to be preferred. Logarithms can be obtained by the aid of the Calculator.

LOGARITHMS.

The value of logarithms to four figures can be obtained from the Calculator by means of Scales Nos. 1 and 5. Limitations of space prevent any lengthy exposition of the theory of logs. but the following notes are useful.

The logarithm of a number is composed of two parts, the Characteristic and the Mantissa.

The **characteristic** is the part of the logarithm to the left of the decimal point, and **may be positive or negative.**

If the number is greater than unity the characteristic is positive and one less in value than the number of figures to the left of the decimal point in the number.

If the number is less than unity the characteristic is negative and one greater than the number of cyphers to the right of the decimal point in the number. The indication of negative value is shown by a minus sign over the top of the characteristic.

The **Mantissa** is the part of the logarithm to the right of the decimal point and is **always positive,** and for the same figures always the same wherever the decimal point may be.

These features of logarithms are shown in the following examples :—

Log of	278	is 2·444.	Log of	0·278	is $\bar{1}{\cdot}444$.
,,	27·8	,, 1·444.	,,	0·0278	,, $\bar{2}{\cdot}444$.
,,	2·78	,, 0·444.	,,	0·00278	,, $\bar{3}{\cdot}444$.

[A fuller description of logs., with tables of logs, and anti-logs., will be found in "Fowler's Machinists Pocket Book," Scientific Publishing Co., Manchester, 4/6 net.]

Ex. 1 : *Find logarithm of* 2675.

Set cursor over 2675 on Scale No. 1. Read Mantissa of log., viz., 427 on scale 5. As there are four figures in the number all to the left of the decimal point, the characteristic of the log. is positive and its valne is 3.

The complete log is 3·427.

Ex. 2 : *Find logarithm of* 50·75.

Set cursor over 5075 on Scale No. 1 (it lies between the second znd third graduation line after 5).

Read Mantissa of log. on Scale 5, viz., 7055.

The characteristic of the log. (as there are two figures to left of decimal point) is 1.

The complete log. is 1·7055.

Ex. 3 : *Find logarithm of* 0·024076.

Set cursor over 24076 on Scale No. 1. This is about one-third of the way between 24 (which represents 240) and the first graduation after it, which represents 2402.

Read mantissa of log. on Scale No. 5.

We make the reading 3815.

As the number is less than unity the characteristic is negative and as there is a cypher to the right of the decimal point its value is 2.

Therefore the logarithm of 0·024076 = $\bar{2}$·3815.

Finding Nth Powers and Nth Roots of Numbers—with logarithms (whether N be a whole number or a fraction).

Let A be a number and suppose $x = A^n$

Where n may be a whole number or a fraction.

Then log. x = n log. A.

Ex. 1 : *Find 5th root of* 51·53 (*i.e. Find* $51{\cdot}53^{\frac{1}{5}}$)

Here n = 1·5th and A = 51·53.

Set cursor over 51·53 on Scale No. 1.

This is between the 3rd and 4th graduations after 5.

Read mantissa of log. on Scale No. 5, viz., 713.

The number is more than unity, therefore the log. is positive. There are two figures to left of decima point, therefore the value of the characteristic is 1.

Therefore the log. of 51·53 = 1·713.

One fifth of log. of 51·53 = 0·3426.

Set cursor over 3426 on Scale No. 1.

Read 5th root of 51·53 on Scale No. 1, viz., 2·2.

Hyperbolic Logarithms.—These which are to the base e = 2·71828 are much used in calculations relating to the expansion of gases. They can be easily derived by multiplying the common logarithm (i.e., the log. to the base 10) by 2·30258.

The exact position of this multiplier denoted by $\log_e 10$ is indicated both on Scale No. 1 and Scale No.

4, but for purpose of finding common logarithms Scale No. 1 must be used.

Ex. 1 : *Find hyperbolic log. of* 14·35.

First find common log. of 14·35.

Set cursor over 1435 on Scale No. 1,

Read mantissa of common log., viz., 1575 on Scale 5. As there are two figures to left of the decimal point in the number and the number is greater than unity, the characteristic is 1 and positive.

Therefore the log. of 14·35 is 1·1575.

Now multiply 1·575 by $\log_e 10$.

Set $\log_e 10$ on Scale No. 1 under datum.

Turn cursor to zero and turn dial till 11575 comes under cursor.

Read answer 266 = hyperbolic log. under datum.

TRIGONOMETRICAL VALUES.

Sines, Tangents, etc.—The values of sines, tangents, etc., are read from the scale of angles No. 6 and No. 7 by means of the cursor.

Read Natural Sin or Natural Tan on Scale No. 1.

Read Log. Sin or Log. Tan on Scale No. 5.

Cosine, Cotangent, Secant and Cosecant are deduced from Sine and Tangent through the following relationships.

For any given angle A :—

$$\text{Cos A} = \text{Sin}\ (90—\text{A})\,; \qquad \text{CotA} = \frac{1}{\text{Tan A}}$$

$$\text{Sec A} = \frac{1}{\text{Cos A}} \qquad \text{Cosec A} = \frac{1}{\text{Sine A}}$$

The Scale of Sines, No. 6, extends twice round the circumference of a circle. The inner circle gives angles between 35 mins. and 5 degs. 45 mins. and the value of the sine increases from 0·01 to 0·10. The outer circle gives angles between 5 degs. 45 mins. and 90 degs., and the value of the Sine increases from 0·10 to 1·0.

Ex. 1 : *Find value of Natural Sine of* 4° 40′.

Set cursor over 4° 40′ on Scale No. 6.

Read Natural sine 0·0813 on Scale No. 1.

N.B.—The number on the scale is 813, but as the sines of all angles on the inner circle of Scale No. 6 are between 0·01 and 0·1 we write down the value 0·0813.

Ex. 2 : *Find value of Natural Sine of* 20° 30′.

Set cursor over 20° 30′ on Scale No. 6.

Read value of Natural Sine 0·3502 on Scale No. 1.

N.B.—The angle being on the outer circle of Scale No. 6, and the angle exceeding 5° 45′, the value of the sine is between 0·1 and 1·0.

Between 20° and 25° the scale is graduated at intervals of 20 so that 20° 30′ falls midway in the second interval following 20°.

In reading the values of log. sines of angles the characteristic of the logs. for all angles between 35 mins. and 5 degs. 45 mins. is 8, and for all angles between 5 deg. 45 mins. and 90 degs. is 9.

The mantissa only of the log. is read on Scale No. 5.

Ex. 3 : Find value of Log. Sine of 27° 20′.

Set cursor over 27° 20′ on Scale No. 6.

Read mantissa of log. sine on Scale No. 5 = 662.

As the angle is on the outer circle of the Sine Scale the log. sine is 9·662.

Ex. 4 : Find value of Log. Sine of 4° 25′.

Set cursor over 4° 25′ on Scale No. 6.

Read mantissa of log. sine on Scale No. 5 = 8865.

As the angle is on the inner circle of the Sine Scale the complete log. sine is 8·8865.

Ex. 5 : Find value of Natural tangent of 31° 45′.

Set cursor over 31° 45′ on Scale No. 7.

Read value of Natural tan. = 0·6188.

Triangles : Useful Notes.—If A, B, C, is a triangle' the angles of which are A, B, C, and the sides opposite these angles are respectively a, b, c.

Then A + B + C = 180°.

$$\frac{a}{b} = \frac{\sin A}{\sin B} \qquad \frac{b}{c} = \frac{\sin B}{\sin C} \qquad \frac{c}{a} = \frac{\sin C}{\sin A}$$

$$a^2 = b^2 + c^2 - 2\,bc \cos A.$$

N.B.—Cos A is itself minus and the whole of the last factor becomes plus if A is greater than 90°.

If A is 90° the last factor disappears.

Sin (180—A) = sin A.

Cos (180—A) = —cos A.

Cotan A = tan (90—A).

$$\text{Area of a traingle} = \frac{a\,b \sin C}{2}$$

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2} \qquad \cot\frac{C}{2} = \tan\frac{A+B}{2}$$

These last two formulae are much used for solving triangles when two sides and included angle are known.

If in a triangle A, B, C, the angle C is a right angle (90°) and the sides opposite the angles are respectively a, b, c (letters arranged clockwise).

$\text{Sin A} = \frac{a}{c}$ $\text{sin B} = \frac{b}{c}$ $\text{cos A} = \frac{b}{c}$ $\text{cos B} = \frac{a}{c}$

$\text{tan A} = \frac{a}{b}$ $\text{cotan A} = \frac{b}{a}$ $\text{secant A} = \frac{a}{b}$

$$\text{cosecant A} = \frac{c}{a}$$

If C is not a right angle the sine, cosine, etc., still have same values, but a and b are not now the sides of the actual triangle, but of an imaginary triangle with B C perpendicular to C A.

A circle is divided into 360 degrees. Each degree is divided into 60′ (minutes); each minute is divided into 60″ (seconds), but seconds are rarely considered.

Multiplication and Division.—The great bulk of calculating work consists of multiplication and division. These operations are in essence addition and subtraction, even in ordinary arithmetic and with the Calculator are reduced to these elementary principles. The working out of a compound fraction, for instance, containing several fractions in the numerator and in the denominator resolves itself into rotating the added numerator factors in one direction, and the subtracted denominator factors in another direction.

Just as there are several ways of working out a compound fraction sum by arithmetic, there are several ways of doing it with a calculator. The numerator factors may be all multiplied together and divided by the total product of the denominator factors, or the factors of the numerator and denominator may be dealt with in pairs, one after the other. Sometimes one method is better than another. The user will discover best methods and short cuts for himself as he acquires proficiency. His first step is to master the principles of operation by studying a few practical examples worked out and described in detail.

Ex. 1 : *Find the product of the factors a, b, c, d,*

Either the Short Scale, No. 1, or the Long Scale, No. 4, may be used. For this illustration Scale No. 1 will be used.

Set factor *a* under datum.
Set cursor to one.
Set dial till factor *b* comes under cursor.
Read product *a* × *b* under datum.
Next set cursor again to one.

Set dial till factor *c* comes under cursor.
Read product $a \times b \times c$ under datum.
Again set cursor to one.
Set dial till factor *d* comes under cursor.
Read product $a \times b \times c \times d$ under datum.

The process after setting the first factor a under the datum is a succession of settings of cursor and of factors on the scale, and of finally reading the product under the datum. The whole operation begins at the datum and ends there. It consists in sum of turning the several factors past a fixed point, and reading the total of angular movements at the end. It matters not whether the angular movements of the dial and cursor are made clockwise or contra-clockwise for the individual settings. So long as the sequence is in the order stated the reading of the final result is the same.

If there are decimal points in the factors the position of the point in the final product is to be decided by inspection and mental consideration as with all logarithmic work. Actual examples of this will be given in the course of the exercises.

If Scale No. 4 (Long Scale) is used instead of Scale No. 1 (Short Scale) the succession of operations is precisely the same, but the setting calls for a little more care as the factors are spread over a scale extending round six circles, and the answer may also be on any one. The particular circle must be determined by a mental consideration of the factors.

The great advantage of the Long-Scale is its fine and uniform graduation, which is in tenths throughout, from end to end. This, with its great length, makes for easy setting and reading coupled with extreme accuracy.

When doing tabular work or working out a series of scientific results the location of the first often serves as a guide to the location of the following ones, the Long Scale then can often be read just as easily as the Short Scale.

To get accustomed to reading Scales Nos. 1 and 4 and their graduations the learner will find it good practice to work through a multiplication table such as twice one are two, twice two are four, etc., thus:

Set 2 on Scale 1 under datum and set cursor to unity

Then turn, in succession, all the figured graduations past the cursor, noting that the procession of values which pass the datum are twice those which pass the cursor.

Thus $2 \times 11 = 22$; $2 \times 12 = 24$; etc

Do the same for 3, or other simple number and proceed to such multipliers as 3·1, etc.

This kind of exercise teaches the learner to read accurately parts of the scale that are not figured or where the graduations require to be split in reading and each counted as 2 if there are 5 graduations, or each counted as 5 if there are 2 graduations.

Division.—This is in essence subtraction, and the reverse of multiplication, which is addition. Its performance with the Calculator is best acquired by practising with simple fractions till the routine of operations becomes mechanical.

Assume the division is of a simple fraction form $\frac{a}{m}$ - i.e., with a single numerator and a single denominator, and also that Scale No. 1 is being used, and the learner is advised to get accustomed to the Scales Nos. 1 and 2 before using Scale No. 4.

Set a under datum.
Set cursor to m.
Set one to cursor.
Read value of $\frac{a}{m}$ under datum.

Scale No. 2 (the Reciprocal Scale graduated anti-clockwise) is often useful for many fractions of the class $\frac{a}{f(x)}$ where the denominator may be x^2, sin. x, tan. x, etc., or any other function read directly on Scale No. 1 and whose reciprocal is at the same time given on Scale No. 2.

Such a fraction then becomes simply the **multiplication** of two factors, viz., $\frac{1}{f(x)}$ and a, and can be done at two settings, but $\frac{1}{f(x)}$ must be set first and then multiplied by the factor (or factors if there are more than one) in the numerator.

This is only one of many devices that can be adopted with the Calculator and make it superior to the straight slide rule.

If the faction is of one of the following forms :—

$$\frac{a \times b \times c}{m} \qquad \frac{a \times b}{m \times n} \qquad \frac{a \times b}{m \times n \times p}$$

where the numerator has not exactly one factor more in it than in the denominator, the question can, as stated, always be worked by multiplying out the numerator and denominator separately and dividing one by the other.

But fractions of this kind, i.e., with multiple factors, are best worked by using up a factor from the top and bottom alternately, and to adopt this method and prevent confusion in operating, the numerator should always contain one more factor than the denominator and to secure this, the artifice of inserting a factor (1) is adopted as often as may be necessary. The above fractions, therefore, before using the Calculator are best changed to the following :—

$$\frac{a \times b \times c}{m \times 1} \qquad \frac{a \times b \times 1}{m \times n} \qquad \frac{a \times b \times 1 \times 1}{m \times n \times p}$$

and worked as follows :—

Taking the faction $\dfrac{a \times b \times c}{m \times 1}$

Set factor a under datum.
Set cursor to m.
Set factor b to cursor.
Set cursor to 1.
Set factor c to cursor.
Read answer under datum.

Taking the faction $\dfrac{a \times b \times 1}{m \times n}$

Set factor a under datum.
Set cursor to m.
Set factor b to cursor.
Set cursor to n.
Set factor 1 to cursor.
Read answer under datum.

Taking the faction $\dfrac{a \times b \times 1 \times 1}{m \times n \times p}$

Set factor a under datum.
Set cursor to m.
Set factor b to cursor.
Set cursor to n.
Set factor 1 to cursor.
Set cursor to p.
Set 1 to cursor.
Read answer under datum.

It will be observed in all these three examples :—

The factors are taken alternately from the numerator and the denominator beginning with the numerator.

The dial is always turned for multipliers.

The cursor is always turned for divisors.

The datum is used only to set first factor and to read final result.

Hints on Arithmetical Calculations: Fixing Decimal Point.—A rough idea of the result of a calculation is often known beforehand, or if not, the position of the decimal point, where necessary, can be approximated by a rough survey of the fraction expressing the required calculation. There are rules, but they are more trouble to remember than they are worth. It is better for the operator to rely on first principles and rapid mental arithmetic.

For example, suppose the value of the following were required :—

$$\frac{6{\cdot}92 \times 746 \times 19{\cdot}2 \times 9}{2876 \times 92{\cdot}5}$$

we could reason mentally, and roughly, as follows :—6·9 is practically 7, and 7 into 2,876 is roughly 400, 400 in 746 is roughly 2: 2 into 92·5 is roughly 45. This would be in the denominator, and for the numerator we should still have left 19·2 × 9, roughly 170. This divided by 45 would obviously give a value less than 10. In putting down the answer, therefore, we should write all figures after the first one to the right of the decimal point. A rough estimate like this occupies less time to make than to describe, and is safer than any cut-and-dried rule.

In simplifying a decimal quantity regard should be paid to the value of any terminal figures struck off.

For example, if we wish to contract a value such a 15·647, then 15·65 is nearer than 15·64, because 7 is nearer 10 than 1. If the number had been 15·642, then 15·64 would have been nearer than 15·65.

A misconception of the fractional value of decimals sometimes causes mistakes, especially if there are cyphers to the right of the decimal point. Remember that when expressed as a fraction, the number of cyphers in the denominator is the same as the number of figures after the decimal point.

For example—

$$3{\cdot}04 = 3\frac{4}{100} \text{ or } \frac{304}{100} \qquad {\cdot}96 = \frac{96}{100} \qquad {\cdot}002 = \frac{2}{1000}$$

In making calculations the quantities should be expressed in proper units. Different kinds of things often required to be multiplied together, but the answer can only have **one quality.** It may be money, weight, force, area, etc. If an area is desired in square feet as a product of linear dimensions, these should be expressed in feet. It does occasionally happen that some quantities are in one unit, and others in another, e.g., in certain formulae for beams lengths are expressed in feet and breadth and depth in inches. Weights of bars are generally given in the per foot run although sectional dimensions are in inches. These points should be borne in mind.

Percentages.—In speaking of percentages, confusion often arises through inattention to the basis on which it is measured. For instance, if A's salary is £75 and B's £50, it would be true to say that A's salary was 50 per cent. greater than B's, taking B's salary as 100 per cent.

It would be equally true to say that B's salary was $33\frac{1}{3}$ per cent. less than A's, taking A's salary as 100 per cent. The fact is only expressed in two different ways.

There can be no misapprehension in any case if the quantity representing the 100 is made clear. Set the question as a problem in fractions, thus :—

Example.—In an examination, 27 scholars pass 1st class; 35, 2nd class; and 63, 3rd class. Express the various numbers as percentages of the whole.

If x, y, z are the three percentages, we have the following relationship :—

$$\frac{27}{125} \times \frac{x}{100} \text{ and } x = \frac{100 \times 27}{125} = 21{\cdot}6 \text{ per cent.}$$

$$\frac{35}{125} = \frac{y}{100} \text{ and } y = \frac{100 \times 35}{125} = 28 \text{ per cent.}$$

$$\frac{\cdot 63}{125} = \frac{z}{100} \text{ and } z = \frac{100 \times 63}{125} = 50{\cdot}4 \text{ per cent.}$$

For this class of question the instrument is very convenient. Set 1·0 on Scale No. 1 under the datum line and the cursor to 125 (i.e., 12·5). Rotate the dial until the several figures 27, 35, 63, come under the Cursor and read the several percentages under the datum.

Problems in Proportion.—Set the queation in simple fractional form as follows where A, B, C are certain known quantities and x is the unknown quantity.

$$\frac{A}{B} = \frac{C}{x}$$

Each of these quantities may be in the numerator or the denominator as the operator finds it convenient to express their relationship but this must, of course be expressed correctly. Then by cross-multiplication we have :—

$$A \times x = B \times C \text{ and } x = \frac{B \times C}{A}$$

Ex. 1 : If 15 men do a task in 28 days, in how many days will 21 men do it, assuming they do it at the same rate ?

Obviously more men will take less time in the ratio of 15 to 21 and if x is the number of days

$$\frac{x}{28} = \frac{15}{21} \text{ and } x = \frac{28 \times 15}{21} = 20 \text{ days.}$$

Ex. 2 : If a task takes 18 men 36 days, how many men will be required to do it in 27 days ?

Obviously more men will be required in proportion to the increased speed at which the task must be done and therefore

$$\frac{36}{27} = \frac{x}{18} \text{ and } x = \frac{36 \times 18}{27} = 24 \text{ men.}$$

Further Use of the Reciprocal Scale.—The introduction of the reversed or reciprocal scale is accompanied by many marked advantages. It corresponds to the reversal of the slider in a straight rule, which is often convenient, though troublesome. In continued multiplication it enables multiplication by two factors to be effected with only one movement of the dial, and compared with the method for finding the product of factors previously described it reduces the number of movements to almost one half.

For example :—Suppose we wish to make a series of multiplications by 3·6, we should proceed as follows :—

1. Set Dial so that Unity line comes under Datum.
2. Turn Cursor to 3·6 on Reciprocal Scale.

Now rotate Dial until all numbers we wish to multiply by 3·6 pass successively under the Cursor, on Scale 1, when their products will be shown successively on this Scale, under the Datum.

Ex. 1.—Multilpying three factors.

Find value of A × B × C.
Set A on Scale 1 under datum.
Set cursor to B on Scale 2.
Set C on Scale 1 under cursor.

Read answer on Scale 1 under datum. (3movements .

Ex. 2.—Multiplying five factors.

Find value of A × B × C × D × E.
Set A on Scale 1 under datum.
Set cursor to B on Scale 2.
Set C on Scale 1 under cursor.
Set cursor to D on Scale 2.
Set E on Scale 1 under cursor.

Read answer on Scale 1 under datum. (5 movements.

Ex. 3.—Operate similarly for any odd number of factors.

It is interesting to compare the above example with the 9 movements necessary when using Scale No. 1. alone or by comparing it with the movements of an ordinary Straight Slide-rule with its intermittent "end-switching." This is only one of many illustrations that could be given.

Ex. 4.—**To Multiply any Even Number of Factors.** —Suppose product of four factors A, B, C, D, or any other even number, is required. It can ve obtained quickly with flcales No. 1 and No. 2 by adding a fector 1 to make the even number of factors into an odd number ,thus :— A × B × C × D × 1. Then proceed exactly as in Example 2 above :—

Set A on Scale 1 under datum.
Set cursor to B on Scale 2.
Set C on Scale 1 under cursor.
Set cursor to D on Scale 2.
Set 1 (unity) under cursor.

Read answer on Scale 1 under datum. (5 movements.)

Rapid Division with Scales No. 1 and No. 2 with Even Number of Factors in the denominator.—

Example 1.—Find value of $\dfrac{A}{B \times C}$

Set A on Scale 1 under datum.
Set Cursor to B on Scale 1.
Set C on Scale 2 under cursor.

Read answers on Scale 1 under datum. (3 movements.)

EXAMPLE 2.—Find value of $\dfrac{A}{B \times C \times D}$

Here the artifice may be adopted of inserting an extra factor 1, into the denominator, to make it contain an even number of factors, thus :—

$$\frac{A}{B \times C \times D \times 1}$$

Then proceed as follows :—

Set A on Scale 1 under datum.
Set cursor to B on Scale 1.
Set C on Scale 2 under cursor.
Set cursor to D on Scale 1.
Set 1 (unity) under cursor.

Read answer on Scale 1 under datum. (5 movements.

Examples of Powers using Reciprocal Scale.

Ex. 1.—Find the value of $(36{\cdot}7)^2$

This can be done with the Calculator in two ways, either by multiplying 36·7 by itself as an ordinary multiplication sum, as previously described, or by the method shown below using the Reciprocal Scale.

Set 36·7 on Scale 1 under Datum.
Set Cursor to 36·7 on Scale 2.
Turn dial till 1 comes under Cursor.
Read 1347 under Datum on Scale 1.

Ex. 2.—Find the value of $(16{\cdot}4)^3$.

This can be done by extended multiplication, 16·4 × 16·4 × 16·4 on Scale 1, or by the method shown below, in which we first find the square of 16·4, as in Example above, and then multiply this result on Scale 1 by 16·4.

Thus set 16·4 on Scale 1 under Datum.
Set Cursor to 16·4 on Scale 2.
Turn dial till 16·4 on Scale 1 comes under Cursor.
Read 441 under Datum on Scale 1.
NOTE.—The result is obtained in 3 movements.

Similarly the use of the Reciprocal Scale makes possible the calculation of expressions such as $\cdot 310^{2\cdot 1}$ or $\cdot 496^{\cdot 5}$ etc., without the cumbersome method of ordinary logarithms, i.e., of having a negative characteristic, which must be made exactly divisible by the index. When decimal quantities are being dealt with, the logarithms are read from the Reciprocal Scale, instead of the outer decimal one.

Additional Notes.—A constant which is very useful is that giving the area of a circle of one inch diameter, expressed in square feet, viz.:—0·785398/144, or 0·00545415. This number, multiplied by A^2, gives at once, in square feet, the area of a circle of diameter A inches.

In regard to Ex. 2 for cube roots it is advisable, when greater accuracy is desired for the root (square or cube) of the reciprocal of a large number, first to find the decimal equivalent of the reciprocal, and then to find the required root of that reciprocal. In this way the last operation (finding the root) tends to reduce any error due to scale reading. In the example as shown on the "reading error" may be increased by the last operation.